33. *On Branched Coverings of Projective Manifolds*

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Introduction. Let M be an n-dimensional complex projective manifold. A *finite branched covering of* M is, by definition, a proper finite holomorphic mapping $\pi: X \to M$ of an irreducible normal complex space X onto M. The *ramification locus* $R_\pi = \{x \in X \mid \pi^*: \mathcal{O}_{M,\pi(x)} \to \mathcal{O}_{X,x}$ is not isomorphic$\}$ of π and the *branch locus* $B_\pi = \pi(R_\pi)$ *of* π are hypersurfaces of X and M, respectively. For a point $x \in \pi^{-1}(B_\pi)$, if $y = \pi(x)$ is a non-singular point of B_π, then x is a non-singular point of both X and $\pi^{-1}(B_\pi)$. In this case, there are coordinate systems $(z_1, \cdots, z_n)$ and $(w_1, \cdots, w_n)$ around x and y, respectively, such that π is locally given by

$$\pi: (z_1, \cdots, z_n) \longmapsto (w_1, \cdots, w_n) = (z_1, \cdots, z_{n-1}, z_n^e).$$

The positive integer e is then locally constant with respect to x. Hence, to every irreducible component D' of $\pi^{-1}(B_\pi)$, a positive integer $e = e_{D'}$ is associated and is called the *ramification index of* π *at* D'. A *covering transformation of* π is an automorphism φ of X such that $\pi\varphi = \pi$. We denote by G_π the group of all covering transformations. π is said to be *Galois* if G_π acts transitively on every fiber of π. π is said to be *abelian* if π is Galois and G_π is an abelian group.

Let $D_1, \cdots, D_s$ be irreducible hypersurfaces of M. Put $B = D_1 \cup \cdots \cup D_s$. Let $e_1, \cdots, e_s$ be positive integers greater than 1. Consider the positive divisor $D = e_1D_1 + \cdots + e_sD_s$. A finite branched covering $\pi: X \to M$ is said to *branch at* D (resp. *at at most* D) if $B_\pi = B$ (resp. $B_\pi \subset B$) and, for every j $(1 \leqslant j \leqslant s)$, and for any irreducible component D' of $\pi^{-1}(D_j)$, the ramification index of π at D' is e_j (resp. divides e_j).

The purpose of this note is (1) to give a criterion for the existence of a finite Galois (resp. abelian) covering of M which branches at D and (2) to describe the set of all (isomorphism classes of) finite Galois (resp. abelian) coverings of M which branch at at most D. We follow the idea of Weil [4].

The detail will be given in Namba [2].

1. Abelian coverings. Let M and D be as above. Consider the additive group

$$\mathrm{Div}\,(M, D) = \{\hat{E} = (a_1/e_1)D_1 + \cdots + (a_s/e_s)D_s + E' \mid a_j \in \mathbf{Z} \text{ for } 1 \leqslant j \leqslant s,\ E' \text{ is an (integral) divisor}\}$$

of rational divisors on M. $E_1, E_2 \in \mathrm{Div}\,(M, D)$ are said to be *linearly equivalent*, $E_1 \sim E_2$, if $E_1 - E_2$ is a principal integral divisor on M. Let