30. *Family of Jacobian Manifolds and Characteristic Classes of Surface Bundles. II*

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1. Introduction. This note is a sequel to our previous papers [2], [3]. There we have investigated cohomological properties of a canonical map, called the Jacobi mapping, from a surface bundle with a cross section to its associated family of Jacobian manifolds and from them we derived new relations among our characteristic classes of surface bundles. The purpose of the present note is to announce new related results. Namely we have obtained still more relations by applying the techniques of [3] to surface bundles *without* cross sections. More precisely in case of a surface bundle with cross section, the structure group of the associated family of Jacobian manifolds was the Siegel modular group $Sp(2g;\boldsymbol{Z})$ which acts on T^{2g} linearly and preserving a prescribed symplectic form ω_0. In the general case we enlarge the structure group to the semi-direct product $T^{2g}\rtimes Sp(2g;\boldsymbol{Z})$. Namely we allow the translations of T^{2g}. The natural action of $T^{2g}\rtimes Sp(2g;\boldsymbol{Z})$ on T^{2g} still preserves the form ω_0. Now we show that for any given surface bundle $\pi: E\to X$, there is a canonical flat T^{2g}-bundle $\pi': J'\to X$ with structure group $T^{2g}\rtimes Sp(2g;\boldsymbol{Z})$ and a natural fibre preserving map $j': E\to J'$ such that the restriction of j' to each fibre induces an isomorphism on the first homology (see Corollary 2). This should be considered as the topological version of Earle's embedding theorem [1] which states that any holomorphic family of compact Riemann surfaces over a complex manifold can be embedded in a certain associated family of Jacobian varieties in an essentially unique way. Earle's family of Jacobian varieties is not the same as the one defined in [3] in general. In fact it may not have any cross section. Moreover even if a surface bundle $\pi: E\to X$ admits a cross section, the flat T^{2g}-bundle $\pi': J'\to X$ above is not in general isomorphic to the previously defined bundle $\pi: J\to X$ ([3]) as *flat* bundles (see § 3). Using this fact we can obtain strong relations among our characteristic classes (see Corollary 6).

2. Topological version of Earle's embedding theorem. Henceforth we use the terminologies of [2], [3] freely. In particular $\mathcal{M}_g$ and $\mathcal{M}_{g,*}$ respectively are the mapping class groups of the closed oriented surface Σ_g of genus $g\geqq 2$ and Σ_g relative to the base point. As in § 6 of [3], we define a crossed homomorphism

$$f_0: \mathcal{M}_{g,*}\times H_1(\Sigma_g)\longrightarrow \boldsymbol{Z}$$