1. A Study of a Certain Non-Conventional Operator of Principal Type. II

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Introduction. We continue our study of the operator

(1) $$B^I = D_t + \sqrt{-1}(t^2/2 + x)D_y,$$

$D_t = -\sqrt{-1}\partial/\partial t$, $D_y = -\sqrt{-1}\partial/\partial y$, in (a neighborhood of the origin in) $\boldsymbol{R}^3$ ([3]). Here we discuss solvability of the equation:

(2) $$B^I u = f$$

for a given f. Since the operator B^I is not locally solvable, our primary task is to specify the conditions on f which guarantee existence of a solution u to (2). One such condition is Condition ($A^{\pm}$) to be introduced in the next section (see also Theorem in § 2).

1. Condition $A^{\pm}$. Let $\beta(t, r, x) = \int_r^t (s^2/2 + x)ds$. Denote by $\tilde{f}$ the Fourier transform of f with respect to the argument y provided it makes sense. Define

(3) $$J^{\pm}(f\,; x, \eta) = \int_{\pm\infty}^{\mp\sqrt{-2x}} \tilde{f}(r, x, \eta) \exp\{\pm\beta(\pm\sqrt{-2x}, r, x)\eta\} dr$$

if $x<0$ and $\pm\eta>0$. Note $\beta(\pm\sqrt{-2x}, r, x)\eta \leqq 0$ in the integrals. We set $J^{\pm}(f\,; x, \eta) = 0$ for $x \geqq 0$ or for $x<0$ and $\pm\eta<0$. We write $J_k^{\pm}(x, \eta)$ instead of $J^{\pm}(f_k\,; x, \eta)$, where $f_k^{\pm} = (t \mp \sqrt{-2x})^k$.

Lemma 1. *For any $x<0$, $\pm\eta>0$ and $m, n = 0, 1, 2, \cdots$, we have*

$$|\partial_\eta^m (x\partial_x)^n J_k^{\pm}(x, \eta)| \leqq C|\eta|^{-(k+1)/3-m}(1+|\eta|(\sqrt{-2x})^3)^{(m+n)/3};$$

$J_0^{\pm}(x, \eta) > 0$ and

$$|\partial_\eta^m (x\partial_x)^n \{J_0^{\pm}(x, \eta)^{-1}\}| \leqq C|\eta|^{1/3-m}(1+|\eta|(\sqrt{-2x})^3)^{1/6+2(m+n)/3}.$$

Here C stands for various constants.

This lemma can be proved by a routine computation. $J_k^{\pm}(x, \eta)$ can be expressed in terms of confluent hypergeometric functions and related functions. For details, see [4].

Now we have to choose the class of functions $f(t, x, y)$ for which the integrals (3) are well-defined. Let $\mathcal{F}$ be the class of distributions $f(t, x, y)$ in $\mathcal{S}'(\boldsymbol{R}^3)$ such that for each $h(y)$ in $\mathcal{S}(\boldsymbol{R}_y)$ the coupling $\langle f(t, x, y), h(y)\rangle$ is continuous in t, at most of polynomial growth in t, and measurable in x. Decompose $f \in \mathcal{F}$ into a difference: $f = f^+ - f^-$, where $\tilde{f}^{\pm}$ are supported in $\pm\eta>0$ so that $f^{\pm}$ have holomorphic extensions in $\pm\operatorname{Im} y>0$. Denote by $\mathcal{F}^{\pm}$ the sets of $f^{\pm}$. Then $\mathcal{F}^{\pm}$ are subspaces of $\mathcal{F}$ and $\mathcal{F} = \mathcal{F}^+ - \mathcal{F}^-$ holds in an obvious manner.

Lemma 2. *Let*

(4) $$(Q^{\pm}f)^{\sim}(x, \eta) = J^{\pm}(f\,; x, \eta)/J_0^{\pm}(x, \eta)$$