27. *Mixed Problem for Weakly Hyperbolic Equations of Second Order with Degenerate First Order Boundary Condition*

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Introduction. In this paper, we are concerned with a mixed problem for second order hyperbolic equations degenerating on the initial surface with degenerate first order boundary condition in $(0, T)\times\Omega$ and prove the existence and uniqueness theorem for classical solutions. The point of our proof is to derive the energy estimate. To do so, we reduce our mixed problem to the one with positive boundary condition for symmetric hyperbolic pseudo differential systems of first order (see [1], [4], [5]). The detailed proof will be given in Tokyo J. Math.

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Writing this paper, we have been informed the result of A. Kubo "On the mixed problems for a weakly hyperbolic equations of second order", which is obtained independently of our paper. Both the result obtained and the method used by him are different from ours.

§1. Statement of the problem and the result. In this paper, we consider the following problem

$$(1.1)\quad \begin{cases} L[u]=\dfrac{\partial^2 u}{\partial t^2}-2t^k\sum\limits_{j=1}^n h_j(t,x)\dfrac{\partial^2 u}{\partial t\partial x_j}-t^{2k}\sum\limits_{i,j=1}^n a_{ij}(t,x)\dfrac{\partial^2 u}{\partial x_i\partial x_j} \\ \qquad +a_0(t,x)\dfrac{\partial u}{\partial t}+t^{k-1}\sum\limits_{j=1}^n a_j(t,x)\dfrac{\partial u}{\partial x_j}+d(t,x)u=f(t,x) \\ u(0,x)=u_0(x), \qquad u_t(0,x)=u_1(x) \\ B[u]|_S=t^k\Big(\sqrt{\sum\limits_{i,j=1}^n a_{ij}(t,s)\nu_i(s)\nu_j(s)}\dfrac{\partial u}{\partial \nu}+\sum\limits_{j=1}^n \alpha_j(t,s)\dfrac{\partial u}{\partial x_j} \\ \qquad -\beta(t,s)\dfrac{\partial u}{\partial t}+\gamma(t,s)u|_S=g(t,s) \end{cases}$$

in the domain $(0, T)\times\Omega$ where Ω is a bounded domain with smooth boundary $\partial\Omega=S$ in $\boldsymbol{R}^n$, k is a positive integer, $\nu(s)=(\nu_1(s),\cdots,\nu_n(s))$ is the inner unit normal at $s\in S$ and $\nu\cdot\alpha=0$. We assume that all the coefficients belong to $\mathscr{B}([0, T]\times\bar{\Omega})$ or $\mathscr{B}([0, T]\times S)$.

For any $s_0\in S$, there is a following smooth coordinate transformation $\Psi: V\to W$ such that