25. A Study of a Certain Non-Conventional Operator of Principal Type

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Introduction. The purpose of this note is to report some of the properties of the first order differential operator:

(1) $$B^I = D_t + i(t^2/2 + x)D_y,$$

$D_t = -i\partial/\partial t$, $D_y = -i\partial/\partial y$, in a neighborhood of the origin in $\boldsymbol{R}^3$. Its principal symbol is given by $b^I = \tau + i(t^2/2 + x)\eta$, if (τ, ξ, η) denotes the dual variables of (t, x, y). Observe then $\{b^I, \bar{b}^I\} = -2it\eta$. Let $S^\pm = \{(t, x, y, \tau, \xi, \eta)\,;\, \tau = 0,\ t^2/2 + x = 0,\ \pm t\eta < 0\}$ and $S_1 = \{(t, x, y, \tau, \xi, \eta)\,;\, \tau = \eta = 0,\ \xi \neq 0\}$. The characteristic set S of B^I is connected and consists of two cones S_1 and $S_2 = S^+ \cup S^- \cup S^0$, where $S^0 = \{(0, 0, y, 0, \xi, \eta)\,;\, \eta \neq 0\}$. A noteworthy fact is that $\{b^I, \bar{b}^I\}/2i$ changes sign on S_2 near S^0. In this sense, the operator B^I does not microlocally fall in the class of operators conventionally studied ([2], [3], [6]). However, we can show the following

Theorem 1. *Let*

(2) $$B^I u = f, \quad u \in \mathscr{D}'(\boldsymbol{R}^3), \quad f \in \mathscr{E}'(\boldsymbol{R}^3),$$

with supp f *in a small neighborhood of the origin. If* $(t_0, x_0, y_0, \tau_0, \xi_0, \eta_0) \in WF(u) \setminus WF(f)$, $\eta_0 \neq 0$, *is in a conic neighborhood* Γ *of* $(0, 0, 0, 0, 0, \eta_0/|\eta_0|)$, *then* $\tau_0 = 0$ *and* $(t_0, x_0, y_0, 0, \xi_0, \eta_0) \in S^+ \cup S^0$.

Note that the general theory [4] assures $WF(u) \setminus WF(f) \subset S$ so that $\tau_0 = 0$ is immediate. A proof of Theorem 1 will be given in §1. We will considerably make use of the particular form of the operator B^I. In this respect, we also include here a result on the equation $B^I u = 0$. Let $u \in \mathscr{D}'(\boldsymbol{R}^3)$. Introduce the quantities:

$$t^*(x, y\,;\, u) = \sup\{t\,;\, (t, x, y) \in \operatorname{supp} u\}, \qquad (x, y) \in \boldsymbol{R}^2,$$
$$y^*(x, t\,;\, u) = \sup\{y\,;\, (t, x, y) \in \operatorname{supp} u\}, \qquad (x, t) \in \boldsymbol{R}^2,$$

adopting the convention $\sup \phi = -\infty$. Replacing sup by inf, we define $t_*(x, y\,;\, u)$ and $y_*(x, t\,;\, u)$ with $\inf \phi = +\infty$. Note $t^*(x, y\,;\, u)$ and $y^*(x, t\,;\, u)$ (resp. $t_*(x, y\,;\, u)$ and $y_*(x, t\,;\, u)$) are upper (resp. lower) semicontinuous.

Theorem 2. *Let* $u \in \mathscr{D}'(\boldsymbol{R}^3)$ *satisfy* $B^I u = 0$. *Assume one of the quantities* $t^*(x, y\,;\, u)$, $y^*(x, t\,;\, u)$, $-t_*(x, y\,;\, u)$ *and* $-y_*(x, t\,;\, u)$ *take a finite local maximum. Then* u *vanishes identically.*

A proof will be given in §2.

Before ending Introduction, we briefly indicate our motivation in