21. *On Ideal Class Groups of Algebraic Number Fields*

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1. Introduction. Nagel [4] proved in 1922 that there exist infinitely many imaginary quadratic number fields with the class numbers divisible by a given natural number. Yamamoto [6] obtained a stronger result for quadratic fields and showed that the same holds also for real quadratic case. On the other hand, Nagel's theorem was extended by Azuhata and Ichimura [1], who constructed, for $m, n\ (>1)$ and $r_2\ (1 \le r_2 \le m/2)$, infinitely many number fields of degree m with just r_2 imaginary prime spots whose ideal class group contains a subgroup isomorphic to $(\boldsymbol{Z}/n\boldsymbol{Z})^{r_2}$. This remarkable result implies the existence of infinitely many number fields of any given degree greater than 1 with the class numbers divisible by any given natural number, but says nothing for totally real number fields.

In this note we extend Yamamoto's theorem to higher degrees. We shall namely show the following

Theorem 1. *For any natural numbers m, n greater than 1 and non negative rational integers r_1, r_2 such that $r_1+2r_2=m$, there exist infinitely many number fields of degree m with just r_1 real (i.e. r_2 imaginary) prime spots whose ideal class group contains a subgroup isomorphic to $(\boldsymbol{Z}/n\boldsymbol{Z})^{r_2+1}$.*

Corollary. *For any natural numbers m, n greater than 1, there exist infinitely many totally real number fields of degree m with the class numbers divisible by n.*

We will now give a brief outline of the proof of the theorem. The details will appear elsewhere.

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2. Notations. We fix, throughout this note, natural numbers m, n greater than 1 and non negative rational integers r_1, r_2 satisfying $r_1+2r_2=m$. Let L be the set of all prime factors of n and put $n_0=\prod_{l \in L} l$.

For a field k, $k^\times$ denotes its multiplicative group and W_k denotes the group of roots of unity contained in k. Note that, if l is prime, then $k^\times/k^{\times l}$ and $k^\times/W_k k^{\times l}$ are both vector spaces over the prime field of characteristic l.