20. The Support of Global Graph Links[*]

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Introduction. In [7], [8], Tamura initiated *global knot theory.* Knots are, by definition, codimension two spheres embedded in manifolds. Tamura calls a knot *local* if it is contained in an embedded ball in the ambient manifold and *unknotted* if it bounds an embedded disk. The following is one of the fundamental problems in global knot theory: *Give criteria for a knot to be local or to be unknotted.* In this note, we study this problem for knots and links in three dimensional manifolds. To avoid the Poincaré conjecture, we need the notion of quasi-localness: A knot is *quasi-local* if it is contained in a homotopy ball embedded in the ambient manifold. Then the fundamental group of the knot exterior determines the quasi-localness and the unknottedness (§ 4).

In contrast with this, if one looks for a criterion for the localness or the quasi-localness of a knot in terms of its homotopy class in the ambient manifold, one encounters the following difficulty: Local knots are *inessential* i.e., null homotopic in the ambient manifold, but the converse does not hold.

In this note we show, however, that if we restrict ourselves to certain graph knots and graph links, the converse does hold. For the precise argument, we introduce the notion of the support of a knot or a link in an orientable three dimensional manifold (§ 1). We then give all the possibility of the support of an inessential graph link (§ 2). Moreover we show that under certain condition on the ambient manifold, the support of a graph link is determined by the homotopy class of each component (§ 3).

This note is an announcement of the author's doctoral thesis at the University of Tokyo. Details will appear elsewhere.

1. Definitions. Manifolds are assumed to be connected, compact, oriented, and of dimension three and links are oriented and may be empty. We say that a manifold is *prime to* $S^1 \times S^2$ if its prime decomposition does not contain $S^1 \times S^2$. A link is *inessential* if each component of it is null homotopic in the ambient manifold and is *nonsplittable* if there does not exist an embedded S^2 in the ambient manifold which separates it into two nonempty sublinks.