19. Representations over G-Rings and Cohomology*)

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(Communicated by Kunihiko KODAIRA, M. J. A., Feb. 13, 1984)

§1. **Introduction.** Let G be a group. The word *ring* will always mean associative ring with an identity element 1. A *G-ring* is a ring Λ together with a G-action on Λ preserving the ring structure. Then we introduce a Grothendieck group $R(G, \Lambda)$ associated with the abelian semi-group consisting of representations over Λ. The group $R(G, \Lambda)$ is a generalization of the representation rings $R(G)$ and $RO(G)$.

The purpose of the present paper is to express $R(G, \Lambda)$ in terms of the cohomology $H^1(G; GL(n, \Lambda))$ of the group G with coefficients in a non-abelian group $GL(n, \Lambda)$ in the sense of Serre [3].

In some cases, $R(G, \Lambda)$ is isomorphic to an equivariant algebraic K-group $K^G(\Lambda; F_f)_d$ and we can express $K^G(\Lambda; F_f)_d$ in terms of the cohomology $H^1(G; GL(n, \Lambda))$. An interesting example is provided by Serre [3]. In fact the example was a starting point of the present investigation.

The consideration of the present paper will be used to prove an induction theorem for equivariant K-theory in a subsequent paper [2].

§2. **$R(G, \Lambda)$.** Let Λ be a G-ring. A *ΛG-module* is a module M over Λ together with a G-action on M such that

$$(*) \qquad g(\lambda_1 m_1 + \lambda_2 m_2) = (g\lambda_1)(gm_1) + (g\lambda_2)(gm_2)$$

for any $g \in G$, $\lambda_i \in \Lambda$, $m_i \in M$. In this paper any modules are assumed to be finitely generated.

Then $R(G, \Lambda)$ is defined to be the abelian group given by generators $[M]$ where M is a ΛG-module which is free over Λ, with relations

$$[M] = [M'] + [M'']$$

whenever $M \cong M' \oplus M''$.

When Λ is a commutative G-ring, we can consider a product $M_1 \otimes M_2$ of two ΛG-modules M_1, M_2 (see [1]). If M_1, M_2 are free over Λ, $M_1 \otimes M_2$ is also free over Λ. Hence this product induces a structure of commutative ring in $R(G, \Lambda)$.

Remark 2.1. If Λ is $\boldsymbol{R}$ (the field of the real numbers) or $\boldsymbol{C}$ (the field of the complex numbers) with trivial G-action, then $R(G, \Lambda)$ is nothing but the real representation ring $RO(G)$ or the complex representation ring $R(G)$ respectively.

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*) Dedicated to Prof. Minoru Nakaoka on his sixtieth birthday. Research supported in part by Grant-in-Aid for Scientific Research.