18. Cohomology mod p of the 4-Connective Fibre Space of the Classifying Space of Classical Lie Groups

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§1. **Introduction.** Let G be a compact, connected, simply connected, simple Lie group. It is well known $\pi_2(G)=0$ and $\pi_3(G)=Z$. Therefore BG, the classifying space of G, is 3-connected and

$$\pi_4(BG)\cong H_4(BG)\cong H^4(BG)\cong Z.$$

Represent a generator x_4 of $H^4(BG)$ by a map $\sigma: BG\to K(Z,4)$ and denote its homotopy fibre by $B\tilde{G}$. Let p be an odd prime and denote the sequence $(p^{k-1},\cdots,p,1)$ by $I(k)$. As is well known

$$H^*(K(Z,3);Z/p)\cong Z/p[\beta\mathcal{P}^{I(k)}u_3;\ k\geq 1]\otimes\Lambda(\mathcal{P}^{I(k)}u_3;\ k\geq 0)$$

where u_3 is a generator of $H^3(K(Z,3);Z/p)\cong Z/p$. The purpose of this paper is to determine $H^*(B\tilde{G};Z/p)$ for any classical type G. The result is

Theorem 1.1. *For any classical type G, there exists an integer $h=h(G,p)$ such that as an algebra*

$$H^*(B\tilde{G};Z/p)\cong H^*(BG;Z/p)/(x_4,\mathcal{P}^{I(1)}x_4,\cdots,\mathcal{P}^{I(h-1)}x_4)\otimes R_h,$$

where R_h is a subalgebra of $H^*(K(Z,3);Z/p)$ generated by $\{\beta\mathcal{P}^{I(k)}u_3;\ k\geq 1\}\cup\{\mathcal{P}^{I(k)}u_3;\ k\geq h\}$. (For $h(G,p)$ see §5.)

The mod 2 cohomology of $B\tilde{G}$ for $G=SU(n)$ or $Sp(n)$ is determined in §4.

§2. **Some algebraic preparations.** Let V be an n-dimensional vector space over F_p. Consider a quadratic form $Q(x)$ on V. It can be thought as an element of degree 2 in $S(V^*)$, the symmetric algebra of the dual space of V. Let $B(x,y)$ be the associated bilinear form of Q (cf. Chap. 4, 1.1 of [5]) and let h be the codimension of the maximal dimensional Q-isotropic subspace of V (cf. Chap. 4, 1.3 of [5]).

Theorem 2.1. *The sequence*

$$(*)\qquad Q(x),\ B(x,x^p),\cdots,B(x,x^{p^{h-1}})$$

is a regular sequence in $S(V^*)$.

For the proof of the above theorem, we look at $\mathrm{Var}\,J$, the algebraic variety defined by J in $V\otimes\Omega$, where J is the ideal of $S(V^*)$ generated by (*) and Ω is an algebraically closed extension of F_p of infinite transcendence degree. In fact

$$\mathrm{Var}\,J=\cup\,W\otimes\Omega$$

where W ranges all maximal Q-isotropic subspaces. Theorem 2.1