17. On the Prolongation of Solutions for Quasilinear Differential Equations

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§0. Introduction. It is well known (M. Zerner [4]) that holomorphic solutions of linear differential equations are holomorphically continued across non-characteristic surfaces. Y. Tsuno [3] showed that this is true for quasilinear equations if the derivatives of solutions up to order $m+1$ are bounded, where m is the order of equation. T. Ishii [1] has recently constructed solutions (for semilinear equations) which are singular along non-characteristic surfaces (see also [2]). So some boundedness conditions are necessary, in general. There is, however, a gap in boundedness properties of solutions between their results.

The aim of this note is to bridge the gap. To do this we show that for each equation there is an exponent σ less than or equal to $m-1$ and that boundedness of order up to σ is sufficient for prolongation.

§1. Definitions. Let Ω be a domain in $\boldsymbol{C}^n$ containing the origin and Λ be the set of multi-indices $\{\beta \in (\boldsymbol{Z}_+)^n : |\beta| \leqq m-1\}$. The variables in $\boldsymbol{C}^n$ and $\boldsymbol{C}^N$ with $N=\sharp\Lambda$ are denoted by $z=(z_1, \cdots, z_n)$ and $p=(p_\beta)_{\beta\in\Lambda}$, respectively. We consider the following quasilinear differential equation:

(1) $$\sum_{|\alpha|=m} a_\alpha(z, (D^\beta u))D^\alpha u = b(z, (D^\beta u)),$$

where $(D^\beta u)=(D^\beta u)_{\beta\in\Lambda}$ with $D=\partial/\partial z$ and $a_\alpha(z, p)$, $b(z, p)$ are holomorphic functions on $\Omega\times\boldsymbol{C}^N$.

Let ϕ be a real-valued C^1 function on Ω with $\phi(0)=0$. We put $\Omega_-=\{z\in\Omega : \phi(z)<0\}$ and $\partial\Omega_-=\{z\in\Omega : \phi(z)=0\}$.

We ask whether u is holomorphic in a neighborhood of the origin if u is holomorphic in Ω_- and satisfies (1).

We assume that $\partial\Omega_-$ is non-characteristic at the origin, that is,

(A) $$\zeta^0=\mathrm{grad}_z\,\phi(0)\neq 0,$$
$$\sum_{|\alpha|=m} a_\alpha(z, p)(\zeta^0)^\alpha \neq 0 \qquad \text{for } (z, p)\in\Omega\times\boldsymbol{C}^N.$$

Under the condition (A) we may assume that $\zeta^0=(1, 0, \cdots, 0)$ and can rewrite (1) as

(2) $$D_1^m u = \sum_{\substack{|\alpha|=m \\ \alpha\neq(m,0,\cdots,0)}} a_\alpha(z, (D^\beta u))D^\alpha u + b(z, (D^\beta u)),$$

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