16. 4-Dimensional Brownian Motion is Recurrent with Positive Capacity

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1. In his pioneering work [1], Fukushima has proved that many sample path properties of the Brownian motion hold except not only on a set of the Wiener measure zero but also on a polar set with respect to the Ornstein-Uhlenbeck process on the Wiener space. Among many results, he proved that for d-dimensional Brownian motion if $d\geq 5$, then the sample paths are transient quasi everywhere, that is, except on a polar set or equivalently except on a set of capacity zero. After him, the author proved as a special case that if $d\leq 3$, the sample paths are recurrent with positive capacity or equivalently the Ornstein-Uhlenbeck process on the Wiener space hit the set of recurrent Brownian paths with positive probability (actually probability 1) [3]. In this paper, we prove that 4-dimensional Brownian paths are also recurrent with positive capacity by taking account of the result of Orey-Pruitt about the N-parameter Wiener process [4].

2. Let $W^{(2,d)}=(W_1^{(2)},\cdots,W_d^{(2)})$ be the 2-parameter Wiener process with values in d-dimensional Euclidean space R^d whose components are independent, that is, each $W_i^{(2)}$, $i=1,\cdots,d$ is an independent copy of a two parameter Gaussian process $\{W(t,s,\omega)\,;\,0\leq t,s<+\infty\}$ defined on a probability space $(\Omega,\mathcal{F},P)$ having continuous sample paths with the mean zero and the covariance

$$E[W(t_1,s_1)W(t_2,s_2)]=(t_1\wedge t_2)(s_1\wedge s_2),$$

where $a\wedge b=\min(a,b)$.

Taking $t\geq 0$ as a parameter set, $W^{(2,d)}(t,\cdot,\omega)\equiv B(t,\omega)$ is considered as a Brownian motion in the sense of Gross [2] with the values in the d-dimensional Wiener space which is a separable Banach space X with a suitable norm. We define the Ornstein-Uhlenbeck process $U(t,\omega)$ as a time change of the Brownian motion by

$$U(t,\omega)=e^{-t/2}B(e^t,\omega). \tag{1}$$

Since X is a subspace of the R^d-valued continuous functions defined on $[0,\infty)$, we denote by $f_s(x)$ for an element x of X the value in R^d at $s\geq 0$.

3. Set

$$A(u,\varepsilon)=\{x\in X\,;\,{}^{\exists}s_n\uparrow+\infty \text{ such that } \|f_{s_n}(x)-u\|<\varepsilon\},$$

where $\varepsilon>0$ and $u\in R^d$, $\|\ \|$ means the usual Euclidean norm in R^d.