# 13. *Stationary Solutions of a Spatially Aggregating Population Model*

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We are concerned with stationary solutions of the following nonlinear degenerate diffusion equation

$$(1)\qquad u_t=(u^m)_{xx}+\left[u\left(\int_{x-r}^{x} u\,dy-\int_{x}^{x+r} u\,dy\right)\right]_x \qquad \text{in } \boldsymbol{R}\times(0,\infty)$$

where $m>1$ is a constant, $0\leqq r\leqq\infty$ a parameter and $u(x,t)\geqq 0$ denotes the population density at position $x\in\boldsymbol{R}$ and time $t>0$. Equations of this type, proposed by Nagai and Mimura [4], represent a spatially spreading population model for a class of aggregating phenomena of individuals. The first term corresponds to the transport of population through a nonlinear diffusion process called density-dependent dispersal (e.g. Gurney and Nisbet [2], Gurtin and MacCamy [3]). The second term provides an aggregative mechanism that moves individuals to the right (resp. left) direction when

$$\int_{x}^{x+r} u(y,t)\,dy>\int_{x-r}^{x} u(y,t)\,dy \qquad (\text{resp. } <).$$

Thus a non-trivial stationary solution of (1) ecologically exhibits an aggregation of individuals.

For a class of Cauchy problems including (1) subject to a nonnegative initial condition $u(x,0)=u^0(x)\geqq 0$ for $x\in\boldsymbol{R}$, Nagai [6] has shown the existence and uniqueness of weak solution. He has also obtained some properties of the solution, for instance, the finite speed of propagation of initial disturbance.

In the cases of $r=0$ and $r=\infty$, stationary solutions of the equation (1) have already been obtained. When $r=0$, (1) is reduced to the porous medium equation (e.g. Aronson [1]) which has no non-trivial stationary solution. In the case of $r=\infty$, Nagai and Mimura [5] have shown that (1) has non-trivial stationary solitary wave solutions.

In the present paper, we deal with the case of $0<r<\infty$, restricting $m=2$. A stationary solution $u(x)$ of (1) with $m=2$ is defined to be a non-negative function belonging to $C(\boldsymbol{R})\cap L_1(\boldsymbol{R})$ that satisfies

(i) $u^2\in C^1(\boldsymbol{R})$,

(ii) $(u^2)_x+u\left(\int_{x-r}^{x} u\,dy-\int_{x}^{x+r} u\,dy\right)=0.$

The main result is described as: