12. *Random Media and Quasi-Classical Limit of Schrödinger Operator*

By Shin OZAWA
Department of Mathematics, University of Tokyo

(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 13, 1984)

In the present note we consider a mathematical problem concerning random media. We consider a bounded domain Ω in $\boldsymbol{R}^3$ with smooth boundary Γ. We put $B(\varepsilon; w)=\{x \in \boldsymbol{R}^3; |x-w|<\varepsilon\}$. Fix $\beta \geq 1$. Let $0<\mu_1(\varepsilon; w(m)) \leq \mu_2(\varepsilon; w(m)) \leq \cdots$ be the eigenvalues of $-\Delta$ $(=-\mathrm{div}\,\mathrm{grad})$ in $\Omega_{\varepsilon, w(m)}=\Omega \backslash \overline{\bigcup_{i=1}^{\tilde{m}} B(\varepsilon; w_i^{(m)})}$ under the Dirichlet condition on its boundary. Here $\tilde{m}$ denotes the largest integer which does not exceed m^β, and $w(m)$ denotes the set of $\tilde{m}$-points $\{w_i^{(m)}\}_{i=1}^{\tilde{m}} \in \Omega^{\tilde{m}}$. Let $V(x)>0$ be C^1-class function on $\bar{\Omega}$ satisfying

$$\int_\Omega V(x)\,dx=1.$$

We consider Ω as the probability space with the probability density $V(x)dx$. Let $\Omega^{\tilde{m}}=\prod_{i=1}^{\tilde{m}} \Omega$ be the probability space with the product measure. The following result which is an elaboration of M. Kac's theorem (Kac [3]) was given in Ozawa [4].

Theorem A. *Assume that* $\beta=1$. *Fix* $\alpha>0$ *and* k. *Then,*

$$\lim_{m\to\infty} \boldsymbol{P}(w(m) \in \Omega^{\tilde{m}}; m^\delta |\mu_k(\alpha/m; w(m))-\mu_k^V|<\varepsilon)=1$$

holds for any $\varepsilon>0$ *and* $\delta \in [0,1/4)$. *Here* μ_k^V *denotes the* k^{th} *eigenvalue of* $-\Delta+4\pi\alpha V(x)$ *in* Ω *under the Dirichlet condition on* Γ.

In this paper we study the case $\beta>1$. In this case the sum of the radii of $\tilde{m}$-balls $B(\alpha/m; w_i^{(m)})$, $i=1, \cdots, \tilde{m}$, tends to ∞ as $m\to\infty$. We see by the argument in Rauch-Taylor [9] that $\mu_k(\alpha/m; w(m))\to\infty$ if $\beta>1$, $V(x)>0$ and

$$\lim_{m\to\infty} \tilde{m}^{-1} \sum_{i=1}^{\tilde{m}} f(w_i^{(m)})=\int_\Omega f(x)V(x)\,dx$$

for any fixed $f \in L^\infty(\Omega)$. We call the case $\beta>1$, $V(x)>0$ to be the solidifying case following Rauch-Taylor.

The aim of this paper is to give the following:

Theorem 1. *Assume that* $\beta \in [1, 9/8)$ *and* $V(x)>0$. *Fix* $\alpha>0$ *and* k. *Then, there exists a constant* $\delta(\beta)>0$ *independent of* m *such that*

$$\lim_{m\to\infty} \boldsymbol{P}(w(m) \in \Omega^{\tilde{m}}; m^{\delta'-(\beta-1)} |\mu_k(\alpha/m; w(m))-\mu_{k,m}^V|<\varepsilon)=1$$

holds for any $\varepsilon>0$ *and* $\delta' \in [0, \delta(\beta))$. *Here* $\mu_{k,m}^V$ *denotes the* k^{th} *eigenvalue of* $-\Delta+4\pi\alpha\tilde{m}m^{-1}V(x)$ *in* Ω *under the Dirichlet condition on* Γ.

Remark. There exist constants C' and C'' such that $C'<m^{-(\beta-1)}\mu_{k,m}^V<C''$ holds.