108. On Semi-Free Unitary S^1-Manifolds

By Masayoshi KAMATA
Department of Mathematics, College of Education,
Kyushu University

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1. Introduction. In [5] C. Kosniowski studied unitary manifolds with an almost effective unitary S^1-action for which the fixed point set is a homology sphere. In this paper we are concerned with unitary S^1-manifolds for which each isotropy group is $\{1\}$ or S^1. These S^1-manifolds are called *semi-free unitary S^1-manifolds*. Let Σ^n be a smooth manifold of the integral homology type of the standard n-sphere S^n. Given an integer k, a homology sphere Σ^{2n} can be equipped with a stable complex structure such that $\tau'(\Sigma^{2n})-\dim_C \tau'=2k\sigma$ if $n\equiv 1 \bmod 4$, and $\tau'(\Sigma^{2n})-\dim_C \tau'=k\sigma$ if $n\equiv 3 \bmod 4$, in the complex K-group $\tilde{K}(\Sigma^{2n})\cong Z(\sigma)$, where τ' is the Whitney sum of the tangent bundle $\tau(\Sigma^{2n})$ and a suitable trivial bundle. The stable complex structures of other spheres are trivial. Let $\Sigma^{2n}(k)$ be the $2n$-sphere with $\tau'(\Sigma^{2n})-\dim_C \tau'=k\sigma$, $n>0$. We then have

Theorem 1. *There exists a semi-free unitary S^1-manifold (M, ϕ) with the fixed point set $\Sigma^{2m}(k_1)+\Sigma^{2m}(k_2)+\cdots+\Sigma^{2m}(k_u)$ such that the normal bundle ν_i of $\Sigma^{2m}(k_i)$ has the m-th Chern class $c^m(\nu_i)=\lambda_i[\Sigma^{2m}(k_i)]$, where $[M]$ indicates the fundamental class of M, if and only if $\sum_{i=1}^u \lambda_i=0$ and $\sum_{i=1}^u k_i=0$.*

The next corollary results from Theorem 1 and the fact that the class $[M]$ of a semi-free S^1-manifold (M, ϕ) in the unitary cobordism group is given by $[M]=\sum_{i=1}^s [P(\nu_i\oplus 1)]$, where the summation is extended over the components $\{F_i\}$ of the fixed point set and $P(\nu_i\oplus 1)$ indicates the projective bundle of $\nu_i\oplus 1$, ν_i the normal bundle of F_i.

Corollary 2. *Let (M, ϕ) be a semi-free unitary S^1-manifold. If the fixed point set is a homology sphere, then M is a boundary.*

The bordism group $F_*^U(S^1)$ of free unitary S^1-manifolds is the free U_*-module with the base $\{[S^{2n+1}, \phi_n]; n=0, 1, 2, \cdots, \phi_n : S^1\times S^{2n+1}\to S^{2n+1}$ is an S^1-action given by $\phi_n(z, v)=zv\}$. By combining the discussion of Theorem 1 with the formal group law theory, we obtain

Theorem 3. *If $\xi\to\Sigma^2$ is an n-dimensional complex vector bundle with $\tau'(\Sigma^2)-\dim_C \tau'=2k\sigma$ in $\tilde{K}(\Sigma^2)\cong Z(\sigma)$, and $c^1(\xi)=\lambda[\Sigma^2]$, then the bordism class $[S(\xi), \phi_\xi]$ of an S^1-action given by $\phi_\xi : S^1\times S(\xi)\to S(\xi)$, $\phi_\xi(z, v)=zv$ is described as follows:*

$$[S(\xi), \phi_\xi]=-\lambda[S^{2n+1}, \phi_n]+(k+\lambda)[CP^1][S^{2n-1}, \phi_{n-1}]$$