107. *Links of Homeomorphisms of Surfaces and Topological Entropy*

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1. Introduction. In [5] the author defined a link L from a given orientation preserving homeomorphism g of a 2-disk D^2 and a finite number of periodic orbits of g in $\mathrm{Int}\, D^2$, and showed that the topological entropy $h(g)$ of g is closely related to the link type of L. In this paper, we extend this result to orientation preserving homeomorphisms of general compact, orientable surfaces. Moreover, we will prove deeper results on periods of periodic orbits of the maps.

In this paper, we suppose that surfaces are connected.

Let F be a compact, orientable surface, f an orientation preserving self-homeomorphism of F and Σ a set consisting of a finite number of periodic orbits of f in $\mathrm{Int}\, F$ with $\chi(F-\Sigma)<0$. Let M_f be the mapping torus of f, i.e. M_f is obtained from $F\times[0, 1]$ by identifying $(x, 0)$ to $(f(x), 1)$ $(x \in F)$. Then $\Sigma\times[0, 1]$ projects to a link $L_{f,\Sigma}$ in M_f. Then our result is:

Theorem. *If $h(f)=0$, then $L_{f,\Sigma}$ is a graph link. Conversely, if $L_{f,\Sigma}$ is a graph link, then f is isotopic rel Σ to a homeomorphism g such that $h(g)=0$. Moreover, if $L_{f,\Sigma}$ is not a graph link and f is differentiable at each point of Σ, then f has infinitely many periodic orbits whose periods are mutually distinct.*

For the definition of graph link see section 2 below.

I would like to express my gratitude to Dr. Koichi Yano for suggesting me to consider this theorem.

2. Preliminaries. A general reference of *topological entropy* is [2, Exposé 10]. In [7] Thurston has shown that if ψ is a homeomorphism of a compact, hyperbolic surface F, then ψ is isotopic to ψ' which is either (1) periodic, (2) pseudo-Anosov, or (3) reducible i.e. there is a system of mutually disjoint simple loops Γ on F such that $\psi'(\Gamma)=\Gamma$; Γ has a ψ'-invariant regular neighborhood $\eta(\Gamma)$ and each ψ'-component of F-$\mathrm{Int}\,\eta(\Gamma)$ satisfies (1) or (2); each component A_i of $\eta(\Gamma)$ is mapped to itself by some positive iterate $(\psi')^{m_i}$ of ψ' and $\psi'^{m_i}|_{A_i}$ is a twist homeomorphism of an annulus [6]. We call ψ' *Thurston's cannonical form* of ψ.

The next assertion was used in the proof of Theorem of [3], but