

106. Invariance of the Plurigenera of Algebraic Varieties under Minimal Model Conjectures

By Noboru NAKAYAMA

Department of Mathematics, University of Tokyo

(Communicated by Kunihiko KODAIRA, M. J. A., Dec. 12, 1984)

The purpose of this paper is to outline our recent results (see Theorems 1, 2 in §3) on the behavior of plurigenera under projective deformation, provided that the minimal model conjectures (see §1) are true. Here all the varieties are defined over the field of complex numbers. Details will be published elsewhere.

§1. Let X be a complete algebraic variety. A divisor D on X is called *nef*, if $D \cdot C \geq 0$ for any curve C on X . The *numerical Kodaira dimension* for a nef Cartier divisor D is defined by

$$\nu(D) := \kappa_{num}(D) := \max \{d \mid D^d \not\equiv 0\}.$$

If $\kappa(D) = \nu(D)$, D is called *good*. If $\kappa(D) = \dim X$ or $\nu(D) = \dim X$, then $\kappa(D) = \nu(D) = \dim X$. Such a D is called *big*.

For a normal variety X , K_X denotes the *canonical divisor class* of X and ω_X denotes the *dualizing sheaf* of X . For a Weil divisor D , if mD is Cartier for some integer m , then D is called *\mathbf{Q} -Cartier*. If K_X is *\mathbf{Q} -Cartier*, then X is called a *\mathbf{Q} -Gorenstein variety*. If any Weil divisors are *\mathbf{Q} -Cartier*, X is called *\mathbf{Q} -factorial*. For a *\mathbf{Q} -Gorenstein variety* X , the smallest positive integer r such that rK_X is Cartier is called the *index* of X , denoted by $\text{index}(X)$. Let X be a normal *\mathbf{Q} -Gorenstein variety*. For some (any) resolution $d: Y \rightarrow X$, if $K_Y = d^*K_X + \sum a_i E_i$, where $\sum E_i$ is a normal crossing d -exceptional divisor, then the singularity of X is called *terminal*, *canonical* or *log-terminal* according as $a_i > 0$, $a_i \geq 0$ or $a_i > -1$, for all i , (see [2] and [5]).

Let X be a normal projective variety with only canonical singularities. For an extremal ray R on X , there exists a morphism $\text{cont}_R: X \rightarrow V$ called a contraction of R . For definitions and details, refer to Kawamata [2]. The type of $\text{cont}_R: X \rightarrow V$ is one of the following cases:

- (i) $\dim X > \dim V$ and cont_R has connected fibers.
- (ii) cont_R is a birational morphism not isomorphic in codimension 1.
- (iii) cont_R is a birational morphism isomorphic in codimension 1.

In the case (i), a general fiber F of the cont_R is a *\mathbf{Q} -Fano variety*, i.e., $-K_F$ is an ample *\mathbf{Q} -Cartier divisor*. In particular, $\kappa(X) = -\infty$. In the case (ii), if X has only *\mathbf{Q} -factorial terminal singularities*, then the