104. *Invariant Polynomials on Compact Complex Manifolds*

By Akito FUTAKI*),†) and Shigeyuki MORITA††)

(Communicated by Kunihiko KODAIRA, M. J. A., Dec. 12, 1984)

1. Introduction. Let M be a compact complex manifold of dimension n, $H(M)$ the complex Lie group of all automorphisms of M and $\mathfrak{h}(M)$ the complex Lie algebra of all holomorphic vector fields of M. In case when $c_1(M)$ is positive, the first author defined in [11] a character $f:\mathfrak{h}(M)\to \boldsymbol{C}$ which is intrinsically defined and vanishes if M admits a Kähler Einstein metric.

The purpose of the present note is twofold. First we generalize the definition of f to obtain a linear map $F: I^{n+k}(GL(n, \boldsymbol{C}))\to I^k(H(M))$ where, for a complex Lie group G, $I^p(G)$ denotes the set of all holomorphic G-invariant symmetric polynomials of degree p. The original f coincides with $F(c_1^{n+1})$ up to a constant. Secondly we give an interpretation of f in terms of secondary characteristic classes of Chern-Simons [9] and Cheeger-Simons [8]. More precisely we show that f appears as the so-called Godbillon-Vey invariant of certain complex foliations.

We also have real analogue of the linear map F for compact group actions. However this case can be derived from the recent papers by Atiyah-Bott [2] and Berline-Vergne [4], which we noticed after we have finished this work. Very interestingly both of the above works [2] [4] are inspired by Duistermaat-Heckman's paper [10] in symplectic geometry while we started from Kählerian geometry. In § 7 we shall derive a Duistermaat-Heckman type formula replacing a symplectic form and a hamiltonian vector fields by a Kähler form and a holomorphic vector field.

2. Definition of $H(M)$-invariant polynomials. Let M be a compact complex manifold of dimension n. Choose any hermitian metric h on M and let D and Θ be the Hermitian connection and the curvature form with respect to h respectively. We put $L(X)=L_X-D_X$ for any $X\in\mathfrak{h}(M)$. For $\phi\in I^{n+k}(GL(n, \boldsymbol{C}))$ we define $f_\phi:(\mathfrak{h}(M))^k\to \boldsymbol{C}$ by

$$f_\phi(X_1, \cdots, X_k)=\int_M \phi\left(L(X_1), \cdots, L(X_k), \frac{i}{2\pi}\Theta, \cdots, \frac{i}{2\pi}\Theta\right).$$

*) Partially supported by JSPS fellowship for research abroad.
†) College of Arts and Sciences, Chiba University.
††) Department of Mathematics, College of Arts and Sciences, University of Tokyo.