103. *On Some Euler Products. II*

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§1. **Meromorphy of Euler products.** Let $E=(P, G, \alpha)$ be an Euler datum in the sense of Part I. We describe a sufficient condition making E and $\bar{E}=(P, G\times \boldsymbol{R}, \bar{\alpha})$ complete when $\mu(P)<d(P)$ $(<\infty)$. We follow the notations of Part I (see [1]).

We say that E satisfies the condition L if E satisfies the following (I)–(III):

(I) $L(s, E, \rho)$ is meromorphic on $\boldsymbol{C}$ for each $\rho\in \mathrm{Irr}^u(G)$.

(II) $L(s, E, \rho)$ is non-zero holomorphic in $\mathrm{Re}(s)\geqq d(P)$ for each $\rho\in \mathrm{Irr}^u(G)$, except for a simple pole at $s=d(P)$ when ρ is trivial.

(III) For each $\rho\in \mathrm{Irr}^u(G)$ and $T>0$, let $S(T, E, \rho)$ be the number of distinct zeros and poles of $L(s, E, \rho)$ in the region $\{s\in \boldsymbol{C};\ 0<\mathrm{Re}(s)\leqq d(P)$ and $-T<\mathrm{Im}(s)<T\}$. Then there exist a positive constant c and a real valued "admissible" function C on $\mathrm{Irr}^u(G)$ such that the following holds:

$$S(T, E, \rho)<C(\rho)(T+1)^c \qquad \text{for all } \rho\in \mathrm{Irr}^u(G) \text{ and } T>0.$$

The admissibility of C is defined as follows. We denote by $\mathrm{Rep}^u(G)$ the set of all equivalence classes of finite dimensional continuous unitary representations of G, which is considered to be a free abelian semigroup (with respect to the direct sum $\oplus$) generated by $\mathrm{Irr}^u(G)$, hence C is naturally considered as a function on $\mathrm{Rep}^u(G)$ by the additive extension. We put $C_0(\rho)=C(\rho)/\deg(\rho)$. We say that C is admissible if there exists a constant $a>0$ such that C_0 satisfies the following (1)–(3):

(1) $C_0(\rho_1\otimes\rho_2)\leqq C_0(\rho_1)+C_0(\rho_2)+a$ for all ρ_1 and ρ_2 in $\mathrm{Rep}^u(G)$;

(2) $C_0(\wedge^j(\rho))\leqq C_0(\rho)j\cdot\deg(\rho)+a$ for all ρ in $\mathrm{Rep}^u(G)$ and $j\geqq 0$, where $\wedge^j(\rho)$ denotes the j-th exterior power of ρ;

(3) $C_0(S^m(\rho))\leqq C_0(\rho)m\cdot\deg(\rho)+a$ for all ρ in $\mathrm{Rep}^u(G)$ and $m\geqq 0$, where $S^m(\rho)$ denotes the m-th symmetric power of ρ.

(For example, deg is an admissible function with any $a\geqq 1$.)

Then we have the following

Theorem 1. *Let $E=(P, G, \alpha)$ be an Euler datum with $\mu(P)<d(P)$. Assume that E satisfies the condition L. Then E and $\bar{E}$ are complete.*

§2. **Note on the proof.** Let G be a topological group. Let $H(T)$ be a polynomial of degree r belonging to $1+T\cdot R^u(G)[T]$. Then, there are continuous functions $\gamma_m:\mathrm{Conj}(G)\to \boldsymbol{C}$ such that