101. *Continuability of Solutions of the Generalized Liénard System with Time Delay*

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1. Introduction. In this paper we consider the system of differential equations

$$
(1.1) \qquad \begin{aligned} x'(t) &= y(t) - F(x(t)) \\ y'(t) &= -g(t, x(t-r(t))) \end{aligned}
$$

where $x'(t)$ and $y'(t)$ denote the right-hand derivatives of x and y at t respectively, and $F: \boldsymbol{R} \to \boldsymbol{R}$, $g: [0, \infty) \times \boldsymbol{R} \to \boldsymbol{R}$, $r: [0, \infty) \to (0, \infty)$ are continuous. Note that other conditions on g, for example $xg(t, x) > 0$ if $x \neq 0$, are not assumed throughout this paper.

Following El'sgol'ts [2], for any $t_0 \geqq 0$, the initial interval at t_0 is given by $E_{t_0} = \{t_0\} \cup \{s : s = t - r(t) < t_0 \text{ for } t \geqq t_0\}$. For any $t_0 \geqq 0$ and any initial function $(\phi, \psi): E_{t_0} \to \boldsymbol{R}^2$, we say $(x(t), y(t))$ is a solution of (1.1) on $[t_0, T)$, where $t_0 < T \leqq \infty$, if $(x(t), y(t))$ is continuous on $E_{t_0} \cup [t_0, T)$ and satisfies (1.1) on (t_0, T) with $(x(t), y(t)) = (\phi(t), \psi(t))$ for all $t \in E_{t_0}$. We denote the solution by $(x(t; t_0, \phi, \psi), y(t; t_0, \phi, \psi))$.

For locally existence of solutions of delay-differential equations we refer the reader to Driver [1] or Hale [3].

The purpose of this paper is to give a necessary and sufficient condition for the continuability of solutions of (1.1).

In [4], Hara, Yoneyama and the author discussed continuation of solutions of the system without time delay

$$
(1.2) \qquad \begin{aligned} x' &= y - F(x) \\ y' &= -g(x) \end{aligned}
$$

and gave some necessary and sufficient conditions under which all solutions of (1.2) are continuable in the future. For example, the following result was given.

Theorem A. *Suppose that*

(i) $\quad xg(x) > 0 \quad$ *if* $|x| > k \quad$ *for some* $k > 0$,

(ii) $\quad \sup\limits_{x \geqq 0} F(x) < \infty \quad$ *and* $\quad \displaystyle\int_0^{\infty} \frac{g(x)}{1 + F_-(x)} dx < \infty$,

(iii) $\quad \inf\limits_{x \leqq 0} F(x) > -\infty \quad$ *and* $\quad \displaystyle\int_0^{-\infty} \frac{g(x)}{1 + F_+(x)} dx < \infty$,

where $F_-(x) = \max\{0, -F(x)\}$ *and* $F_+(x) = \max\{0, F(x)\}$. *Then all solutions of* (1.2) *are continuable in the future if and only if*