100. A Note on Sums and Maxima of Independent, Identically Distributed Random Variables

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§1. **Introduction.** Let $X_1, X_2, \cdots$ be i.i.d. (independent, identically distributed) random variables and put $S_n = X_1 + \cdots + X_n$, and $M_n = \max(X_1, \cdots, X_n)$ $n = 1, 2, \cdots$. Chow-Teugels [2] studied the joint limiting distributions of (S_n, M_n) as $n \to \infty$ after suitable normalizations. In this note we will consider this problem using the theory of point processes and generalize the result of [2] to a functional limit theorem for the sums and the maxima of triangular arrays of i.i.d. random variables.

§2. **Main theorem.** Let $\{\xi_{nk}\}_{k=1}^{\infty}$ be i.i.d. random variables with distribution function $F_n(x)$, $n=1, 2, \cdots$. Throughout this paper we assume that for suitably chosen constants A_n, $n=1, 2, \cdots$ and a nondegenerate distribution function $F(x)$ we have

(2.1) $\lim_{n\to\infty} P[\sum_{j=1}^{n} \xi_{nj} - A_n \leqq x] = F(x)$ at all continuity points of $F(x)$.

The characteristic function $\phi(\theta)$ of $dF(x)$ has the following representation.

$$\phi(\theta) = \exp\left[\gamma\theta - \frac{1}{2}\sigma^2\theta^2 + \int \{e^{i\theta x} - 1 - i\theta x I(|x| \leqq \delta)\}\mu(dx)\right] \tag{2.2}$$

where $\gamma \in \boldsymbol{R}$, $\sigma^2 \geqq 0$, $\int \min(1, x^2)\mu(dx) < \infty$ and $\delta > 0$ is chosen so that $\mu\{\pm\delta\} = 0$.

It is well known that (2.1) implies a functional limit theorem; the process

$$\xi_n(t) = \textstyle\sum_{j \leqq nt} \xi_{nj} - A_n[nt]/n \tag{2.3}$$

converges in law to the Lévy process $\xi(t)$ with characteristic (2.2) over the Skorohod function space $D([0, \infty); \boldsymbol{R})$ endowed with the J_1-topology (see [5] for the definition). We also assume that there exist constants $B_n > 0$, C_n, $n=1, 2, \cdots$ and nondegenerate distribution function $G(x)$ such that

(2.4) $\lim_{n\to\infty} P[B_n \max_{k \leqq n} \xi_{nk} - C_n \leqq x] = G(x)$ at all continuity points of $G(x)$ (see Lemma 4.1.).

(As we will see later, if $\mu(0, \infty) > 0$ then this condition is automatic from (2.1) with $B_n = 1$, $C_n = 0$.) It is also well known that (2.4) implies a functional limit theorem: Define