99. *On a Semilinear Diffusion Equation on a Riemannian Manifold and its Stable Equilibrium Solutions*

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§ 1. **Introduction.** Let (M, g) be a connected orientable compact C^∞ Riemannian manifold with (possibly empty) smooth boundary ∂M.

We consider the following semilinear diffusion equation and its equilibrium solutions.

$$\frac{\partial u}{\partial t}=\Delta u+f(u) \qquad \text{in } (0, \infty)\times M \tag{1.1}$$

$$\frac{\partial u}{\partial \nu}=0 \qquad \text{on } (0, \infty)\times \partial M \tag{1.2}$$

where f is a smooth function on $\boldsymbol{R}$ into $\boldsymbol{R}$, $\Delta=\text{div grad}$ is the Laplace-Beltrami operator with respect to the metric g and ν denotes the outward unit normal vector on ∂M. In the case $\partial M=\phi$, we eliminate (1.2).

In this note, we will report that the system (1.1)–(1.2) does not admit any spatially inhomogeneous stable equilibrium solution under some geometrical assumptions for M, while it is not the case with some (M, g) and f.

In the case that M is a bounded domain in the Euclidean space, Matano has proved in [4] that if the domain is convex, then any stable equilibrium solution must be a constant function, and he has also constructed a domain and a function f for which the system (1.1)–(1.2) admits a non-constant stable equilibrium solution. Then our result may be regarded as a generalization of his result to the case of manifolds.

§ 2. **Statement of the results.**

Theorem 1. *Assume the following conditions* (1) *and* (2):

(1) *M has non-negative Ricci curvature,* i.e. *for any $x \in M$ and $X \in T_xM$, $R(X, X)\geqq 0$ holds. Here $R(\cdot, \cdot)$ denotes the Ricci tensor.*

(2) *The second fundamental form of ∂M with respect to ν in M is non-positive definite.*

Then any non-constant equilibrium solution of (1.1)–(1.2) *is unstable.*

Remark 1. In the case $\partial M=\phi$, we eliminate the assumption (2) in Theorem 1.

Remark 2. If M is a bounded subdomain of $\boldsymbol{R}^n$ with smooth