87. *The Global Hypoellipticity of a class of Degenerate Elliptic-Parabolic Operators*

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There are vast references on *local* hypoellipticity of degenerate elliptic-parabolic operators (cf. Amano [1], [2], Fedii [3], Hörmander [5], Morimoto [8], Oleinik and Radkevich [9] and their references), however one can find only few papers concerned with *global* hypoellipticity. Oleinik and Radkevich [9] and Kusuoka and Stroock [7] gave several criteria of global hypoellipticity. Fujiwara and Omori [4] found an operator which is globally hypoelliptic but not locally hypoelliptic. In this note, we give a criterion of global hypoellipticity which is finer than Oleinik and Radkevich's result, and show a theorem as one of its applications. Fujiwara and Omori's result is contained in our theorem as a special case. Oleinik and Radkevich's and Kusuoka and Stroock's theorems are not applicable to the operators treated in our theorem. We can apply our criterion to the wider class of degenerate elliptic-parabolic operators.

Let P be a differential operator of the form

$$P=\partial_1^2+a(x)\partial_2^2$$

with nonnegative coefficient $a(x)$ in $C^\infty(\boldsymbol{T}^2)$, where $\partial_i=\partial/\partial x_i$ $(i=1,2)$ and $\boldsymbol{T}^2$ is the 2-dimensional torus $\boldsymbol{R}^2/2\pi\boldsymbol{Z}^2$. X_0, X_1, X_2 denote vector fields defined by

$$X_0=-(\partial_2 a(x))\partial_2,\quad X_1=\partial_1,\quad X_2=a(x)\partial_2$$

and S is a subset of $\boldsymbol{T}^2$ defined by

$$S=\{x\in\boldsymbol{T}^2 : \dim \mathrm{Lie}\,(x)<2\},$$

where $\mathrm{Lie}\,(x)=\{X(x) : X\in \mathrm{Lie}\,(X_0, X_1, X_2)\}$.

Theorem. *Assume that*

(1) $$\partial_2^2 a(x)=0 \qquad on\ S.$$

Then the operator

$$P=\partial_1^2+a(x)\partial_2^2$$

is globally hypoelliptic in $\boldsymbol{T}^2$ *if and only if the system*

(2) $$\dot{x}=\sum_{i=0}^{2}\xi_i X_i(x), \qquad \xi_i\in\boldsymbol{R}$$

is controllable in $\boldsymbol{T}^2$.

Remark. The author does not know whether or not Theorem is valid without the assumption (1). It is to be noted that Theorem remains valid in case the set S and its boundary ∂S are nonsmooth.