

## 74. Dualizing with respect to $s$ -tuples

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**1. Introduction.** In a projective plane, if the roles of lines and points are interchanged, the dual geometry is obtained. Similar concept was introduced in a field of the design of experiments by Bose and Nair [1], who derived a new class of designs, by interchanging blocks and treatments in a given class of block designs. This concept of interchanging the roles of blocks and treatments is usually named as "DUALIZATION". We denote the dual of the design  $D$  by  $D_1^*$ . This dualization, that is, writing the block numbers of blocks in which a treatment occurs in the original design, is extended to another concept as writing the block numbers of blocks in which a pair of treatments occurs in the original design. This is named as "Dualization with respect to (w.r.t.) pairs", which is denoted by  $D_2^*$  for a given block design  $D$ , and is dealt with in Mohan and Kageyama [6]. In this note, the concept of dualization w.r.t. pairs is further generalized in the form as "Dualization w.r.t.  $s$ -tuples" for  $s \geq 1$ . This dual design is denoted by  $D_s^*$ . Applying this technique to certain designs yields new block designs  $D_s^*$  for some values of  $s$ . For the description of some technical terms in designs, we refer the reader to Raghavarao [7].

**2. Method.** We here consider an equireplicated and equiblock-sized design in which the number of treatments (with the replication number  $r$ ) is  $v$  and the number of blocks (of size  $k$ ) is  $b$ . The present method is as follows: Number the blocks of a given block design  $D$ . Now in  $D_s^*$  if the  $i$ -th block of  $D$  includes an  $s$ -tuple, then the corresponding block of  $D_s^*$  will have the  $i$ -th treatment of  $D_s^*$ . This  $D_s^*$  coincides with the known cases described in the introduction when  $s=1$  and 2.

For a given block design  $D$  with parameters  $v, b, r$  and  $k$ , it is obvious that its dual design  $D_s^*$  w.r.t.  $s$ -tuples, for  $s < k$ , is characterized by the parameters in the following form:

$$v' = b, \quad b' = \binom{v}{s}, \quad r' = \binom{k}{s},$$

$k'$  = the number of times  $s$ -tuples of treatments occur in the original design,

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