73. On Characters of Irreducible Highest Weight Representations of Witt Algebra

By Shuichi SUGA
Department of Mathematics, Faculty of Science, Osaka University

(Communicated by Shokichi IYANAGA, M. J. A., Sept. 12, 1984)

1. Introduction. Many results on characters of irreducible highest weight representations of Witt algebra were obtained by several authors (V. G. Kac [3], [4] and A. Rocha-Caridi and N. R. Wallach [6]). In this paper we determine the remaining characters by using the methods of [6].

The *Witt algebra* is an infinite dimensional complex Lie algebra with basis $\{E_i\}_{i\in Z}$ which have the following commutation relations:

$$[E_i, E_j]=(j-i)E_{i+j} \qquad i, j\in Z.$$

It is also known as a Lie algebra of polynomial vector fields on the circle. Let us denote the Witt algebra by $\mathfrak{g}$.

A *highest weight module* of $\mathfrak{g}$ is defined as follows.

Definition. A $\mathfrak{g}$-module M is called the *highest weight module* with highest weight $\lambda\in C$ if there exists a nonzero vector v such that

(1) $E_i\cdot v=0$ for $i>0$

(2) $E_0\cdot v=\lambda v$

(3) M is generated by v as $\mathfrak{g}$-module.

If M is a highest weight module with highest weight λ, then M is decomposed as a direct sum of its weight spaces relative to the action of E_0:

$$M=\oplus_{i=0}^{\infty} M_{\lambda-i}$$

where $M_{\lambda-i}=\{u\in M\,;\, E_0\cdot u=(\lambda-i)u\}$.

We define the formal character of M by

$$\mathrm{ch}\, M=\textstyle\sum_{\nu\in C}(\dim M_{-\nu})e^{\nu}$$

where e^{ν} is a formal exponential.

For any complex number λ there exists a unique irreducible highest weight module $L(\lambda)$ with highest weight λ.

Our main theorem is the following.

Theorem. *Put* $\lambda_m=-(m^2-1)/24$ *for nonnegative integer* m.

(a) *For* $\lambda=\lambda_m$, $m\equiv 2 \pmod 6$, *we have*

$$\mathrm{ch}\, L(\lambda)=e^{-\lambda}\phi(e)^{-1}(1-e^{2(m+4)/3}).$$

(b) *For* $\lambda=\lambda_m$, $m\equiv 4 \pmod 6$, *we have*

$$\mathrm{ch}\, L(\lambda)=e^{-\lambda}\phi(e)^{-1}(1-e^{(m+2)/3}).$$

where $\phi(e)=\prod_{i=1}^{\infty}(1-e^i)$ *is the generating function of the classical partition function.*