8. *Exponential Quadratic Splines*

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1. Introduction and consistency relations. In practical applications, curves and surfaces which appear as smooth as possible to the human viewer are to be fitted through given points in a plane or in space, so it has been common practice to restrict attention to splines which are piecewise polynomials with continuous first or second derivatives. However, certain generalized splines are sometimes more useful as they permit the variation of additional parameters. In this regard, various results have been already obtained on exponential cubic splines and the other generalized cubic splines (Späth [4]).

In the present note, we consider exponential quadratic splines and their consistency relations among function or derivative values at mesh and mid points.

Let Δ_n be a partition of the interval [0, 1] with the following mesh points:

(1.1) $$\Delta_n : 0=x_0<x_1<\cdots<x_n=1,$$

and $\Lambda=(\lambda_0, \lambda_1, \cdots, \lambda_{n-1})$ be a system of n non-negative numbers. Then we define an exponential quadratic spline $s(x) \in C^1[0,1]$ associated to (Δ_n, Λ) as follows:

(1.2) $$s(x)=\alpha_i+\beta_i\phi_i\{(x-x_i)/h_i\}+\gamma_i\psi_i\{(x-x_i)/h_i\}$$
$$x_i \le x \le x_{i+1}, \quad h_i=x_{i+1}-x_i, \quad i=0, 1, \cdots, n-1$$

where for $\lambda_i>0$

(1.3) (i) $\phi_i(x)=(1/\lambda_i)\sinh(\lambda_i x)$
(ii) $\psi_i(x)=(2/\lambda_i^2)\{\cosh(\lambda_i x)-1\}$ ([5]),

and for $\lambda_i=0$, $\varphi_i(x)=x$ and $\psi_i(x)=x^2$.

By simple calculation, we have

(1.4) $$\phi_i(x)\longrightarrow x \quad \text{and} \quad \psi_i(x)\longrightarrow x^2 \quad \text{as} \quad \lambda_i\longrightarrow 0.$$

Since $s(x)$ depends upon six parameters $\alpha_j, \beta_j, \gamma_j$, $j=i, i+1$ on $[x_i, x_{i+2}]$ and continuity conditions of $s^{(k)}(x)$, $k=0, 1$ at $x=x_{i+1}$ gives us two conditions toward the determination of these parameters, five quantities s_j, $j=i, i+1, i+2$ and $s_{j+1/2}$, $j=i, i+1$ must be interrelated, i.e., we have

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