72. *Fourier Coefficients of Eisenstein Series of Degree 3*

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Our aim is to give explicit formulas of Eisenstein series for $Sp_3(\boldsymbol{Z})$ except the 2-Euler factor. In case of $Sp_2(\boldsymbol{Z})$ they are given in [5] and essentially in [1]. It is known, [6], that they are products of local densities of quadratic forms up to elementary factors. Thus we have only to evaluate local densities.

In this note, we assume that p is an odd prime, and $m(\geqq 4)$ is even natural number. Set

$$S=\begin{pmatrix} & 1_{m/2} \\ 1_{m/2} & \end{pmatrix} \qquad (1_{m/2}=\text{the identity matrix of degee } m/2).$$

We use the notation $\alpha_p(T,S)$ in [2] for local densities, and for simplicity we write $\alpha(T)$ for $\alpha_p(T,S)$. Denote by χ the quadratic residue symbol mod p.

Theorem. *Set* $d=(1-p^{-m/2})(1-p^{2-m})$, *and for a diagonal matrix* T *whose diagonal entries are* $\varepsilon_i p^{a_i}$ $(1\leq i\leq 3)$ *with* $\varepsilon_i\in \boldsymbol{Z}_p^\times$, $-1\leq a_1\leq a_2\leq a_3$, *set*

$$\gamma(T)=\alpha(p^2T)-(p^{3-m/2}+p^{5-m})\alpha(pT)+p^{8-3m/2}\alpha(T),$$

and $\chi(T)=1$, $\chi(-\varepsilon_1\varepsilon_2)$, $\chi(-\varepsilon_2\varepsilon_3)$ *or* $\chi(-\varepsilon_1\varepsilon_3)$ *according to* $a_1\equiv a_2\equiv a_3 \bmod 2$, $a_1\equiv a_2\not\equiv a_3 \bmod 2$, $a_1\not\equiv a_2\equiv a_3 \bmod 2$ *or* $a_1\not\equiv a_2\not\equiv a_3 \bmod 2$. *Then we have*

$$\gamma(T)/d=1+\chi(T)p^{(2-m/2)(a_1+a_2+a_3+6)}, \qquad \textit{and}$$

1) *in case* $a_1\equiv a_2 \bmod 2$,

$$\begin{aligned}\alpha(T)/d=&\sum_{0\leq k\leq a_1}\Big(\sum_{0\leq i\leq (a_1+a_2)/2-k-1}p^{(5-m)i}\Big)p^{(3-m/2)k}\\&+p^{a_1/2+(5-m)a_2/2}\Big(\sum_{0\leq k\leq a_1}p^{(2-m/2)k}\Big)\Big(\sum_{0\leq j\leq[(a_3-a_2-1)/2]}p^{(4-m)j}\Big)\\&+\chi(-\varepsilon_1\varepsilon_2)p^{a_1/2+(5-m)a_2/2}\Big(\sum_{1\leq k\leq a_1+1}p^{(2-m/2)k}\Big)\Big(\sum_{0\leq j\leq[(a_3-a_2)/2-1]}p^{(4-m)j}\Big)\\&+\chi(T)p^{(a_1+a_2)/2+(2-m/2)a_3}\Big(\sum_{0\leq k\leq a_1}p^{(2-m/2)k}\Big)\Big(\sum_{0\leq j\leq (a_2-a_1)/2}p^{(3-m)j}\Big)\\&+\chi(T)p^{(m/2-1)a_1+(2-m/2)(a_2+a_3)+3-m}\sum_{0\leq k\leq a_1-1}\Big(\sum_{0\leq j\leq k}p^{(1-m/2)j}\Big)p^{(2-m/2)k},\end{aligned}$$

2) *in case* $a_1\not\equiv a_2 \bmod 2$,

$$\begin{aligned}\alpha(T)/d=&\sum_{0\leq k\leq a_1}\Big(\sum_{0\leq j\leq (a_1+a_2-1)/2-k}p^{(5-m)j}\Big)p^{(3-m/2)k}\\&+\chi(T)p^{(m/2-1)a_1+(2-m/2)(a_2+a_3)+3-m}\sum_{0\leq k\leq a_1-1}\Big(\sum_{0\leq j\leq k}p^{(1-m/2)j}\Big)p^{(2-m/2)k}\\&+\chi(T)p^{(a_1+a_2)/2+(2-m/2)a_3+(3-m)/2}\Big(\sum_{0\leq k\leq a_1}p^{(2-m/2)k}\Big)\Big(\sum_{0\leq j\leq (a_2-a_1-1)/2}p^{(3-m)j}\Big).\end{aligned}$$

Corollary 1. *Let* $a_k(T)$ *be the Fourier coefficient of Eisenstein series of weight* k $(\equiv 0 \bmod 2)$ *for* $Sp_n(\boldsymbol{Z})$ $(n\leq 3)$. *Let* T *be a half integral positive definite* $n\times n$ *matrix. Then the Dirichlet series*