60. *Confluent Hypergeometric Functions on an Exceptional Domain*

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(Communicated by Shokichi IYANAGA, M. J. A., June 12, 1984)

In [3], G. Shimura studied the generalized confluent hypergeometric functions on tube domains of several types. A motive of his study can be seen in the application to the Eisenstein series as developed in his recent paper [4]. In this paper, we shall describe analogous results in the case of tube domains constructed from Cayley's octonion (which includes the case of exceptional simple tube domain).

We denote by $\mathfrak{C}_{\boldsymbol{R}}$ the real Cayley algebra, and we fix the standard basis (e.g. cf. [2]). For each integer m $(1\leqq m\leqq 3)$, we put $\kappa(m)=4m-3$. We define a vector space $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ over $\boldsymbol{R}$ by $\mathfrak{J}_{\boldsymbol{R}}^{(m)}=\{x\in \boldsymbol{M}_m(\mathfrak{C}_{\boldsymbol{R}})\,|\,{}^t\bar{x}=x\}$, where the bar denotes the Cayley conjugation. We supply $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ with a product by $x\circ y=(1/2)(xy+yx)$, with this product, $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ becomes a real Jordan algebra. When $m=3$, $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ is called the exceptional Jordan algebra (cf. [1]). If $x=(x_{ij})\in\mathfrak{J}_{\boldsymbol{R}}^{(m)}$, we define $\mathrm{tr}\,(x)=\sum x_{ii}\in\boldsymbol{R}$ and define an inner product $(\ ,\ )$ on $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ by $(x,y)=\mathrm{tr}\,(x\circ y)$. Moreover, we define a polynomial function det on $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ as follows. When $m=3$,

$$\det(x)=\prod_{i=1}^{3}x_{ii}-x_{11}\boldsymbol{N}(x_{23})-x_{22}\boldsymbol{N}(x_{13})-x_{33}\boldsymbol{N}(x_{12})+\boldsymbol{T}((x_{12}x_{23})\bar{x}_{13}),$$

where $\boldsymbol{N}(a)=a\bar{a}=\bar{a}a$, $\boldsymbol{T}(a)=a+\bar{a}$ $(a\in\mathfrak{C}_{\boldsymbol{R}})$. In the case $m=2$, we define as $\det(x)=x_{11}x_{22}-\boldsymbol{N}(x_{12})$. We denote by $\mathfrak{R}_m$ the set of squares $x\circ x$ of elements of $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$, and by $\mathfrak{R}_m^+$, the interior of $\mathfrak{R}_m$; then $\mathfrak{R}_m^+$ is a convex open cone in $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$. $\mathfrak{R}_3^+$ is called the exceptional cone. Identifying $\boldsymbol{C}^{m\kappa(m)}$ with $\mathfrak{J}_{\boldsymbol{C}}^{(m)}=\mathfrak{J}_{\boldsymbol{R}}^{(m)}\otimes_{\boldsymbol{R}}\boldsymbol{C}$, we define a tube domain $\boldsymbol{H}_m$ by $\boldsymbol{H}_m=\{x+iy\,|\,x\in\mathfrak{J}_{\boldsymbol{R}}^{(m)},\ y\in\mathfrak{R}_m^+\}$. Then $\boldsymbol{H}_3$ is the exceptional tube domain of type E_7 (cf. [1]) and $\boldsymbol{H}_1$ is the complex upper-half plane. We define a Euclidean measure dx on $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ by viewing $\mathfrak{J}_{\boldsymbol{R}}^{(m)}$ as $\boldsymbol{R}^{m\kappa(m)}$. Now we define the generalized gamma function $\Gamma_m(s)$ associated with the cone $\mathfrak{R}_m^+$ by

$$\Gamma_m(s)=\int_{\mathfrak{R}_m^+}e^{-\mathrm{tr}\,(x)}\det(x)^{s-\kappa(m)}dx,$$

then the integral converges for $\mathrm{Re}\,(s)>\kappa(m)-1$ and satisfies the following identity:

$$\Gamma_m(s)=\pi^{2m(m-1)}\prod_{n=0}^{m-1}\Gamma(s-4n),$$

where $\Gamma(s)$ is the ordinary gamma function (e.g. cf. [1]). Put, for $g\in\mathfrak{R}_m^+$, $h\in\mathfrak{J}_{\boldsymbol{R}}^{(m)}$, and $(\alpha,\beta)\in\boldsymbol{C}^2$,