48. *On the Resolution of Two-dimensional Singularities*

By Mutsuo OKA
Department of Mathematics, Faculty of Sciences,
Tokyo Institute of Technology

(Communicated by Kunihiko KODAIRA, M. J. A., May 12, 1984)

§1. **Introduction.** Let $f(z_1, \cdots, z_n)$ be a germ of an analytic function at the origin such that $f(0)=0$ and f has an isolated critical point at the origin. We assume that f has a non-degenerate Newton boundary. Let V be a germ of hypersurface $f^{-1}(0)$. Let $\Gamma^*(f)$ be the dual Newton diagram and let Σ^* be a simplicial subdivision of $\Gamma^*(f)$. It is known that there is a canonical resolution $\pi: \tilde{V} \to V$ which is associated with Σ^*. ([1]). However the process to get Σ^* from $\Gamma^*(f)$ is not unique and a "bad" Σ^* gives unnecessary exceptional divisors. The purpose of this paper is to show that in the case $n=3$, there is a canonical subdivision Σ^* of $\Gamma^*(f)$ so that the resolution graph is obtained by a canonical surgery from $S_2\Gamma^*(f)$ (=two-skeleton of $\Gamma^*(f)$). See Theorem (5.1).

§2. **Newton boundary and the dual Newton diagram.** Let $f(z_1, \cdots, z_n)=\sum_\nu a_\nu z^\nu$ be the Taylor expansion of f where $z^\nu = z_1^{\nu_1} \cdots z_n^{\nu_n}$. Recall that the Newton boundary $\Gamma(f)$ is the union of the compact faces of $\Gamma_+(f)$ where $\Gamma_+(f)$ is the convex hull of the union of the subsets $\{\nu + (\boldsymbol{R}^+)^n\}$ for ν such that $a_\nu \neq 0$. For any closed face Δ of $\Gamma(f)$, we associate the polynomial $f_\Delta(z)=\sum_{\nu \in \Delta} a_\nu z^\nu$. We say that f is *non-degenerate* if f_Δ has no critical point in $(\boldsymbol{C}^*)^n$ for any $\Delta \in \Gamma(f)$ ([2]).

Let N^+ be the space of positive vectors in the dual space of $\boldsymbol{R}^n$. For any vector $P={}^t(p_1, \cdots, p_n)$ of N^+, we associate the linear function $P(x)=\sum_i p_i x_i$ on $\Gamma_+(f)$ and let $d(P)$ be the minimal value of $P(x)$ on $\Gamma_+(f)$ and let $\Delta(P)=\{x \in \Gamma_+(f)\,;\, P(x)=d(P)\}$. We introduce an equivalence relation $\sim$ on N^+ by $P \sim Q$ if and only if $\Delta(P)=\Delta(Q)$. For any face Δ of $\Gamma_+(f)$, let $\Delta^*=\{P \in N^+\,;\, \Delta(P)=\Delta\}$. The collection of Δ^* gives a polyhedral decomposition of N^+ which we call *the dual Newton diagram of f* and we denote it by $\Gamma^*(f)$. $\Delta(P)$ is a compact face of $\Gamma(f)$ if and only if P is strictly positive. We say that a subdivision Σ^* of $\Gamma^*(f)$ is a *simplicial subdivision* if the following conditions are satisfied ([1]).

(i) Σ^* is a subdivision by the cones over a simplicial polyhedron whose simplexes are spanned by primitive integral vectors with determinant ± 1 in the sense of §3.

(ii) Let $\sigma=(P_1, \cdots, P_n)$ be an $(n-1)$-simplex. Then there exists