47. Vanishing Theorems in Asymptotic Analysis. II

By Hideyuki MAJIMA
Department of Mathematics, Hitotsubashi University

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Let M be a complex manifold and let H be a divisor on M. For simplicity, suppose that the divisor H has only normal crossings. Denote by $\mathcal{O}$ the sheaf of germs of holomorphic functions, by $\mathcal{O}(*H)$ the sheaf of germs of meromorphic functions which are holomorphic in M–H and have poles on H and by $\mathcal{O}_{M\hat{|}H}$ the formal completion of $\mathcal{O}$ along H: $\mathcal{O}_{M\hat{|}H}=\text{Proj}\lim_{k\to\infty}\mathcal{O}/\mathcal{J}_H^k$, where $\mathcal{J}_H$ is the nullstellen ideal of H. Let M^- be the real blowing up of M along H and let $pr: M^-\to M$ be the natural projection. Denote by $\mathcal{A}^-$ the sheaf of germs of functions strongly asymptotically developable, and denote by $\mathcal{A}'^-$ and $\mathcal{A}_0^-$ the sheaf of germs of functions strongly asymptotically developable to $\mathcal{O}_{M\hat{|}H}$ and to 0, respectively. Then, we have the short exact sequence of sheaves:

$$0\longrightarrow\mathcal{A}_0^-\xrightarrow{i}\mathcal{A}'^-\xrightarrow{FA}pr^*(\mathcal{O}_{M\hat{|}H})\longrightarrow 0,$$

from which we obtain the long exact sequence of cohomologies:

$$\begin{aligned}0&\longrightarrow H^0(p^-,\mathcal{A}_0^-|_{p^-})\xrightarrow{i_{p,0}}H^0(p^-,\mathcal{A}'^-|_{p^-})\longrightarrow H^0(p^-,pr^*(\mathcal{O}_{M\hat{|}H})|_{p^-})\\&\longrightarrow H^1(p^-,\mathcal{A}_0^-|_{p^-})\xrightarrow{i_{p,1}}H^1(p^-,\mathcal{A}'^-|_{p^-})\longrightarrow H^1(p^-,pr^*(\mathcal{O}_{M\hat{|}H})|_{p^-})\\&\longrightarrow H^2(p^-,\mathcal{A}_0^-|_{p^-})\longrightarrow\cdots\longrightarrow H^n(p^-,\mathcal{A}_0^-|_{p^-})\longrightarrow\cdots,\end{aligned}$$

where $p^-=pr^{-1}(p)$, and

$$\begin{aligned}0&\longrightarrow H^0(M^-,\mathcal{A}_0^-)\xrightarrow{i_0}H^0(M^-,\mathcal{A}'^-)\longrightarrow H^0(M^-,pr^*(\mathcal{O}_{M\hat{|}H}))\\&\longrightarrow H^1(M^-,\mathcal{A}_0^-)\xrightarrow{i_1}H^1(M^-,\mathcal{A}'^-)\longrightarrow H^1(M^-,pr^*(\mathcal{O}_{M\hat{|}H}))\\&\longrightarrow H^2(M^-,\mathcal{A}_0^-)\longrightarrow\cdots\longrightarrow H^n(M^-,\mathcal{A}_0^-)\longrightarrow\cdots.\end{aligned}$$

In the previous article [2], we assert that $i_{p,1}$ is a zero mapping for any point p in H, and that i_1 is a zero mapping if $H^1(M,\mathcal{O})=0$. Here, we assert moreover the following:

Theorem 1. *For any point p in H, $H^q(p^-,\mathcal{A}_0^-|_{p^-})=0$, $q=2,\cdots,n$. If $H^q(M,\mathcal{O})=0$ and $H^q(M,\mathcal{O}_{M\hat{|}H})=0$, $q=1,\cdots,n$, then $H^q(M^-,\mathcal{A}_0^-)=0$, $q=2,\cdots,n$.*

In order to prove Theorem 1, we use the following soft resolution of the sheaf $\mathcal{A}_0^-$:

$$\mathcal{A}_0^-\longrightarrow\mathcal{P}_{0,0}^-\xrightarrow{d''}\mathcal{P}_{0,1}^-\xrightarrow{d''}\cdots\xrightarrow{d''}\mathcal{P}_{0,n}^-\xrightarrow{d''}0,$$

where $\mathcal{P}_{0,q}^-$ denotes the sheaf on M^- of germs of differential forms of type $(0,q)$ with coefficients infinitely differentiable and infinitely flat on $pr^{-1}(H)$. Notice that the direct image $pr_*(\mathcal{P}_{0,q}^-)$ on M coincides