45. On the Rank of Hasse-Witt Matrix*)

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1. Let A be an algebraic function field of one variable with a perfect field K of characteristic $p\neq 0$ as the exact constant field. Let D be the K-module of differentials of A. Let G, E^* and R be the K-submodules of differentials of the first kind, of pseudo-exact differentials and of residue free differentials in D, respectively.

The following equality was proven by the author [2], and by Kunz [4] in the case where K is algebraically closed:

$$\dim_K R/E^* = \dim_K G/G\cap E^*.$$

The author proved in [3] that this equality still holds true and the both dimensions are unchanged by any algebraic constant field extension of A over K.

Let M be the Hasse-Witt matrix (identified with the Cartier-Manin matrix) of A over K with respect to a basis of G. Then we shall show

Proposition. *We have* $\operatorname{rank}(M^{(p^{1-g})}\cdots M^{(p^{-1})}M) = \dim_K G/G\cap E^*$, *where* $g>0$ *is the genus of* A *and each* $M^{(p^{-j})}$ *is the matrix of* p^{-j}*-th power raised elements of* M.

Corollary 1. *The* p*-rank of the null class group of* $A\bar{K}$, *the constant field extension of* A *by the algebraic closure* $\bar{K}$ *over* K, *is equal to* $\dim_K G/G\cap E^*$.

Corollary 2. $M^{(p^{1-g})}\cdots M^{(p^{-1})}M=0$ *holds if and only if* $G\subseteqq E^*$.

Corollary 3. *We have* $\operatorname{rank}(M^{(p^{1-g})}\cdots M^{(p^{-1})}M) = \dim_K R/E^*$.

2. Let A^p be the subfield of p-power elements of A. If x is in $A\setminus A^p$, then $\{1, x, \cdots, x^{p-1}\}$ is a basis of A over A^p, and any ω of D is representable in such form as

$$\omega = \sum_{j=0}^{p-1} a_j^p x^j dx.$$

Then the Cartier operator C is defined by $C(\omega)=a_{p-1}dx$. The following properties are well-known (see [1]);

(1) C is independent of a choice of x.

(2) $C(y_1^p\omega_1+y_2^p\omega_2)=y_1C(\omega_1)+y_2C(\omega_2)$ for $y_1, y_2\in A$ and $\omega_1, \omega_2\in D$.

(3) $C(\omega)$ is in G if ω is in G.

Let us denote by E_n the K-submodule of ω of D with $C^n(\omega)=0$. $E_{n+1}\supseteqq E_n$ for every n is evident. Let us define $E^*=\bigcup_{n=1}^{\infty} E_n$ and call the elements of E^* *pseudo-exact differentials*. In particular, we call the elements of E_1 *exact differentials*.

*) Dedicated to Professor Kentaro Murata on his 60th birthday.