41. Local Existence of C^∞-Solution for the Initial-Boundary Value Problem of Fully Nonlinear Wave Equation

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We shall consider the local existence in time of C^∞-solutions for the following initial-boundary value problem:

$$\text{(M.P)} \qquad \begin{aligned} &\mathcal{L}u+F(t,x,\bar{D}^2u)=f(t,x) \quad \text{in } [0,T]\times\Omega,\\ &u=0 \quad \text{on } [0,T]\times\partial\Omega,\\ &u(0,x)=\psi_0(x),\quad (\partial_t u)(0,x)=\psi_1(x) \quad \text{in } \Omega,\end{aligned}$$

where

$$\mathcal{L}v=\partial_t^2 v+a_1(t,x,\bar{D}_x^1)\partial_t v+a_2(t,x,\bar{D}_x^2)v,$$
$$a_1(t,x,\bar{D}_x^1)v=\textstyle\sum_{j=1}^n a_2^j(t,x)\partial_j v+a_1^0(t,x)v,$$
$$a_2(t,x,\bar{D}_x^2)v=-\textstyle\sum_{i,j=1}^n a_2^{ij}(t,x)\partial_i\partial_j v+\sum_{j=1}^n a_1^j(t,x)\partial_j v+a_0(t,x)v,$$

and $a_2(t,x,\bar{D}_x^2)$ is a strictly elliptic operator with $a_2^{ij}=a_2^{ji}$. Here and hereafter we use the notations:

$$\partial_t=\partial_0=\partial/\partial t,\quad \partial_j=\partial/\partial x_j,\quad \partial_x^\alpha=\partial_1^{\alpha_1}\cdots\partial_n^{\alpha_n}\quad (|\alpha|=\alpha_1+\cdots+\alpha_n),$$

and for any integer $L\geqq 0$

$$D^L v=(\partial_t^j\partial_x^\alpha v\,;\,j+|\alpha|=L),\qquad \bar{D}^L v=(\partial_t^j\partial_x^\alpha v\,;\,j+|\alpha|\leqq L),$$
$$D_x^L v=(\partial_x^\alpha v\,;\,|\alpha|=L),\qquad \bar{D}_x^L v=(\partial_x^\alpha v\,;\,|\alpha|\leqq L).$$

Ω is a domain in $\boldsymbol{R}^n$ with compact and C^∞ boundary $\partial\Omega$. Let T be some positive constant.

In the case of $\Omega=\boldsymbol{R}^n$ the local existence in time of C^∞-solutions of fully nonlinear wave equations is already known (see, e.g., [2]), since we can reduce fully nonlinear equations to quasilinear systems by the method due to Dionne [1], the local solvability of which has extensively been studied (see, e.g., Kato [3] and [4]). However, we can not apply that method to the initial-boundary value problem. Accordingly, for the initial-boundary value problem the Nash-Moser technique has often been used in order to overcome the so-called derivative loss which results from the fully nonlinearity of the equation (see, e.g., [5], [7], [9], [10] and [11]). Moreover, because of the difficulty of the derivative loss it has been unknown whether the C^∞-solution exists or not even when $\psi_0(x)$, $\psi_1(x)$ and $f(t,x)$ are in a class of C^∞.

In the present paper we give the local existence theorem of C^∞-solutions of Problem (M.P). Our method is essentially based on the

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