39. *Structure of the Solution Space of Witten's Gauge-field Equations*

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§0. **Introduction.** Consider a gauge field in the eight-dimensional space satisfying

$$(1)\qquad \begin{aligned}&[\nabla_{y_\mu}, \nabla_{y_\nu}] = (1/2)\varepsilon_{\mu\nu\alpha\beta}[\nabla_{y_\alpha}, \nabla_{y_\beta}],\\ &[\nabla_{z_\mu}, \nabla_{z_\nu}] = (-1/2)\varepsilon_{\mu\nu\alpha\beta}[\nabla_{z_\alpha}, \nabla_{z_\beta}],\\ &[\nabla_{y_\mu}, \nabla_{z_\nu}] = 0, \qquad (\mu, \nu = 0, 1, 2, 3)\end{aligned}$$

where $(y, z) = (y_0, y_1, y_2, y_3, z_0, z_1, z_2, z_3) \in \boldsymbol{C}^8$, Δ_{y_μ} etc. are covariant derivatives, and $\varepsilon_{\mu\nu\alpha\beta}$ is the totally anti-symmetric tensor such that $\varepsilon_{0,1,2,3} = 1$.

Set $x = (y+z)/2$, $w = (y-z)/2$. E. Witten [1] pointed out that (1) implies the second-order Yang-Mills equations

$$(2)\qquad [\nabla_{x_\mu}, [\nabla_{x_\mu}, \nabla_{x_\nu}]] = 0 \qquad (\nu = 0, 1, 2, 3)$$

on the diagonal subspace $\Delta = \{(y, z) \mid w = 0\}$, and further, that a gauge field on Δ satisfies (2) if and only if it can be extended to a neighborhood of Δ consistently to (1) mod $(w_0, w_1, w_2, w_3)^2$. Here $(w_0, w_1, w_2, w_3)^2$ denotes the square of the ideal generated by w_0, w_1, w_2, w_3.

In this paper, we rewrite (1) in the language of Sato's soliton theory [2] and investigate the structure of the solution space of (1) on the analogy of Takasaki's work ([3], [4]): we solve an initial-value problem of differential equations with respect to functions with value in an infinite-dimensional Grassmann manifold. (See Theorem 2.)

In our case, there appear a pair of spectral parameters λ_1, λ_2. The main difference from the case of one spectral parameter is that the initial data must satisfy a system of differential equations if the problem is solvable. (See Proposition 4 and cf. [3].)

§1. **Linearization.** Set $\eta_1 = y_0 + iy_1$, $\zeta_1 = y_2 - iy_3$, $\eta_2 = z_0 + iz_1$, $\zeta_2 = z_2 - iz_3$, $\bar\eta_1 = y_0 - iy_1$, $\bar\zeta_1 = y_2 + iy_3$, $\bar\eta_2 = z_0 - iz_1$, and $\bar\zeta_2 = z_2 + iz_3$. Then, introducing parameters λ_1, λ_2, we can rewrite (1) as follows: for any $\lambda_1, \lambda_2 \in \boldsymbol{C}$,

$$(3)\qquad \begin{aligned}&[-\lambda_1\nabla_{\eta_1} + \nabla_{\zeta_1}, \lambda_1\nabla_{\zeta_1} + \nabla_{\bar\eta_1}] = 0, \qquad [-\lambda_2\nabla_{\eta_2} + \nabla_{\zeta_2}, \lambda_2\nabla_{\bar\zeta_2} + \nabla_{\bar\eta_2}] = 0,\\ &[-\lambda_1\nabla_{\eta_1} + \nabla_{\zeta_1}, -\lambda_2\nabla_{\eta_2} + \nabla_{\zeta_2}] = [-\lambda_1\nabla_{\eta_1} + \nabla_{\zeta_1}, \lambda_2\nabla_{\bar\zeta_2} + \nabla_{\bar\eta_2}] = 0,\\ &[\lambda_1\nabla_{\bar\zeta_1} + \nabla_{\bar\eta_1}, -\lambda_2\nabla_{\eta_2} + \nabla_{\zeta_2}] = [\lambda_1\nabla_{\bar\zeta_1} + \nabla_{\bar\eta_1}, \lambda_2\nabla_{\bar\zeta_2} + \nabla_{\bar\eta_2}] = 0.\end{aligned}$$

Throughout this paper we discuss in the category of formal power series, so that $\nabla_{\eta_1} = \partial_{\eta_1} + A_{\eta_1}$, $A_{\eta_1} \in \mathrm{gl}(n, \boldsymbol{C}[[\eta_1, \zeta_1, \cdots, \bar\zeta_2]])$ etc.

Now we "fix" the gauge, namely, restrict the freedom of gauge so that $A_{\eta_1} = A_{\zeta_1} = A_{\eta_2} = A_{\zeta_2} = 0$. Then (3) reads