38. On Pluricanonical Maps for 3-Folds of General Type

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The purpose of this note is to outline our recent result on the pluricanonical maps for nonsingular projective 3-folds of general type. Details will be published elsewhere.

Let X be a nonsingular projective 3-dimensional variety over the complex number field, which is called a "3-fold" in short. The canonical divisor K_X is said to be "nef" if the intersection number $K_X \cdot C \geqq 0$ for any curve C on X. Moreover, K_X is said to be "big" if $\kappa(K_X, X) = \dim X$ (cf. Iitaka [6] and Reid [10]), i.e., if X is of general type. For any n with $h^0(X, \mathcal{O}_X(nK_X)) \neq 0$, we have the n-ple canonical linear system $|nK_X|$ and associated with this, we have the rational map $\Phi_{|nK_X|}$.

Main Theorem. *Let X be a nonsingular projective 3-fold whose canonical divisor K_X is nef and big.*

Then

(i) *$\Phi_{|7K_X|}$ is birational with the possible exceptions of*

a) *$\chi(\mathcal{O}_X)=0$ and $K_X^3=2$, or*

b) *$|3K_X|$ is composed of pencils,*

i.e., $\dim \Phi_{|3K_X|}(X)=1$,

(ii) *$\Phi_{|nK_X|}$ is birational for $n \geqq 8$.*

Corollary. *Assume further that K_X is ample. Then $\Phi_{|nK_X|}$ is birational for $n \geqq 7$.*

The hypothesis that K_X is ample is required only to derive the inequality

$$\chi(\mathcal{O}_X) < 0 \qquad \text{(cf. Yau [11]).}$$

There is a conjecture that this inequality holds even when K_X is nef and big. Therefore, once this conjecture is established, we will have a sharper result that $\Phi_{|nK_X|}$ is birational for $n \geqq 7$ whenever K_X is nef and big.

X. Benveniste announced in [2] the same result as our main theorem. But his proof is incomplete. Modifying his argument, we can complete the proof and get a better result when K_X is ample.

§1. The following theorem about a surface plays a crucial role in our proof of the main theorem. We replace the condition $h^0(S, \mathcal{O}_S(mR)) \geqq 7$ in Proposition 2-0 of Benveniste [1] by (*) below, which is weaker than the former.