34. A Shape of Eigenfunction of the Laplacian under Singular Variation of Domains. II

—The Neumann Boundary Condition—

By Shin OZAWA
Department of Mathematics, University of Tokyo

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Let Ω be a bounded domain in $\boldsymbol{R}^2$ with smooth boundary γ. Let B_ε be the ε-ball whose center is $w \in \Omega$. We put $\Omega_\varepsilon = \Omega \setminus \bar{B}_\varepsilon$. We consider the following eigenvalue problem:

$$(1)\qquad \begin{aligned} -\Delta_x u(x) &= \lambda(\varepsilon) u(x), && x \in \Omega_\varepsilon \\ u(x) &= 0, && x \in \gamma \\ \frac{\partial u}{\partial \nu}(x) &= 0, && x \in \partial B_\varepsilon, \end{aligned}$$

where $\partial/\partial\nu$ denotes the derivative along the inner normal vector at x with respect to the domain Ω_ε. Let $0 < \mu_1(\varepsilon) \leq \mu_2(\varepsilon) \leq \cdots$ be the eigenvalues of (1). Let $0 < \mu_1 \leq \mu_2 \leq \cdots$ be the eigenvalues of $-\Delta$ in Ω under the Dirichlet condition on γ. We arrange them repeatedly according to their multiplicities. Let $\{\varphi_j(\varepsilon)\}_{j=1}^\infty$ (resp. $\{\varphi_j\}_{j=1}^\infty$) be a complete orthonomal basis of $L^2(\Omega_\varepsilon)$ (resp. $L^2(\Omega)$) consisting of $-\Delta$ eigenfunctions of associated with $\{\mu_j(\varepsilon)\}_{j=1}^\infty$ (resp. $\{\mu_j\}_{j=1}^\infty$).

We assume that w is the origin of $\boldsymbol{R}^2$. We use the polar coordinates $z - w = (r\cos\theta, r\sin\theta)$. The aim of this note is to give the following:

Theorem 1. *Fix j. Assume that μ_j is a simple eigenvalue. Let ρ be an arbitrary fixed positive number. Then,*

$$(3)\qquad \|\varphi_j(\varepsilon) - t_\varepsilon \varphi_j\|_{L^\infty(\Omega_\varepsilon)} = O(\varepsilon^{1-\rho})$$

and

$$(4)\qquad \left(\left(\frac{\partial}{\partial\theta}(\varphi_j(\varepsilon))\right)(\varepsilon\cos\theta, \varepsilon\sin\theta)\right) = 2t_\varepsilon(\partial_{\vec{wz}}\varphi_j(w)|_{w=0}) + O(\varepsilon^{1-\rho})$$

hold, where $\partial_{\vec{wz}}\varphi_j(w)$ denotes the derivative of $\varphi_j(w)$ with respect to w along the vector $\overrightarrow{wz}$. Here

$$s_\varepsilon = \int_{\Omega_\varepsilon} (\varphi_j(\varepsilon))(x)\varphi_j(x)dx, \qquad t_\varepsilon = \operatorname{sgn} s_\varepsilon.$$

Remarks. The remainders in (3), (4) are not uniform with respect to j. We can prove that s_ε^2 tends to 1 as $\varepsilon \to 0$. The relationship between Theorem 1 and the following Theorem A in Ozawa [2] was discussed in Ozawa [2]. The Hadamard variational formula (see Garabedian-Schiffer [1]) plays an essential role in their relationship.