33. *On the Confluent Euler-Poisson-Darboux Equation and the Toda Equation*

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§1. **Summary.** The Toda equation with two time variables

(1.1) $$XY \log t_n = t_{n+1}t_{n-1}/t_n^2 \qquad \left(X=\frac{\partial}{\partial x},\ Y=\frac{\partial}{\partial y},\ t_n=t_n(x,y)\right)$$

can be solved using solutions of the confluent Euler-Poisson-Darboux equation

(1.2) $$(XY+xX+\alpha-n)u_n=0.$$

Rational solutions, confluent hypergeometric solutions and solutions which can be expressed by hypergeometric functions with two variables are obtained.

§2. **Bäcklund transformation of a separated solution.** As is shown in our previous work ([1])

(2.1) $$t_n=F(n)\exp((\alpha-n)xy)$$

where $F(n+1)F(n-1)/F(n)^2=\alpha-n$, $F(0)=F(1)=1$, satisfies the Toda equation (1.1).

(2.2) $$r_n=XY\log t_n=\alpha-n, \qquad s_n=Y\log t_{n-1}/t_n=x$$

satisfies

(2.3) $$Yr_n=r_n(s_n-s_{n+1}), \qquad Xs_n=r_{n-1}-r_n.$$

Put

(2.4) $$M_n=XY+s_{n+1}X+r_n=XY+xX+\alpha-n,$$
$$X_n=-r_n^{-1}X=(n-\alpha)^{-1}X, \qquad Y_n=Y+s_{n+1}=Y+x.$$

Define

(2.5) $$T=\{u_n=u_n(\alpha\,;x,y)\,;\,M_0u_0=0,\ u_{n+1}=Y_nu_n\ (n\geqslant 0),$$
$$u_{n-1}=X_nu_n\ (n\leqslant 0)\}$$

then we have

Theorem 2.1 (Bäcklund transformation). *If* $u_n\in T$ *then we have* $M_nu_n=0$, $u_{n+1}=Y_nu_n$, $u_{n-1}=X_nu_n$ $(n=0,\pm 1,\pm 2,\cdots)$ *and* $\tau_n=u_nt_n$ *satisfies the Toda equation* (1.1).

§3. **One-parameter groups on *T*.** We can obtain three linearly independent first order partial differential operators which commute with M_0 (modulo M_0).

Theorem 3.1. $\hat{X}=X+y$, Y *and* $Z=yY-xX$ *commute with* M_0.

We can construct three one-parameter groups of linear transformations and a finite group which keep T invariant.