32. On the Growth of Meromorphic Solutions of an Algebraic Differential Equation

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1. Introduction. In 1933 Yosida ([14]) applied the Nevanlinna theory of meromorphic functions to differential equations in the complex plane for the first time and generalized a Malmquist's theorem ([7]).

Theorem of Yosida. *If the differential equation*
(1) $(w')^m = R(z, w)$, *R rational in z, w and m a positive integer, possesses a transcendental meromorphic solution $w = w(z)$ in the complex plane, then $R(z, w)$ must be a polynomial in w of degree at most $2m$. Further, if $w(z)$ has only a finite number of poles, the degree is at most m.*

Later various mathematicians studied differential equations in the complex plane with the aid of Nevanlinna theory (see the references in [1], [13]) and many generalizations of this theorem have been obtained by several authors ([2], [5], [6], [11], [12], etc.).

In this paper we shall consider a general differential equation studied in [2], [6], [11] and [12]. We denote by $\mathcal{M}$ the set of meromorphic functions in the complex plane and by $\mathcal{L}$ the set of $E \subset [0, \infty)$ for which means $E < \infty$. Further, the term "meromorphic" will mean meromorphic in the complex plane.

Let P be a polynomial of $w, w', \cdots, w^{(n)}$ $(n \geqq 1)$ with coefficients in $\mathcal{M}$:

$$P(z, w, w', \cdots, w^{(n)}) = \sum_{\lambda \in I} c_\lambda(z) w^{i_0} (w')^{i_1} \cdots (w^{(n)})^{i_n},$$

where $c_\lambda \in \mathcal{M}$ and I is a finite set of multi-indices $\lambda = (i_0, i_1, \cdots, i_n)$ for which $c_\lambda \neq 0$ and $i_0, i_1, \cdots, i_n$ are non-negative integers, and let $A(z, w)$, $B(z, w)$ be polynomials in w with coefficients in $\mathcal{M}$ and mutually prime in $\mathcal{M}$:

$$A(z, w) = \sum_{j=0}^{p} a_j(z) w^j, \qquad B(z, w) = \sum_{k=0}^{q} b_k(z) w^k,$$

where $a_j, b_k \in \mathcal{M}$ such that $a_p \cdot b_q \neq 0$.

We shall consider the differential equation

$$(2) \qquad P(z, w, w', \cdots, w^{(n)}) = A(z, w)/B(z, w).$$

We put

$$\Delta = \max_{\lambda \in I} (i_0 + 2i_1 + \cdots + (n+1)i_n),$$
$$d = \max_{\lambda \in I} (i_0 + i_1 + \cdots + i_n),$$
$$\Delta_o = \max_{\lambda \in I} (i_1 + 2i_2 + \cdots + ni_n).$$