28. *On n-Unitary Subsemigroups of Semigroups*

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Let S denote a semigroup and H a subset of S. Using notation $H\cdots a=\{(x, y)\in S\times S: xay\in H\}$ for all elements a in S, it can be easily verified that $P_H=\{(a, b)\in S\times S: H\cdots a=H\cdots b\}$ is a congruence on S. P_H is called the principal congruence on S determined by H ([1]).

In [2] it is shown that if H is a reflexive unitary subsemigroup of a semigroup S, then S/P_H is either a group or a group with zero. Conversely, if P is a congruence on a semigroup S such that S/P is a group or a group with zero, with identity H, then H is a reflexive unitary subsemigroup of S and $P_H=P$ (Theorem 1.1 of [2]).

In [2] we also proved that if H and N are unitary subsemigroups of a semigroup S such that H is reflexive in S, then $H\cap N$ is either empty or a reflexive unitary subsemigroup of N and $\langle H, N\rangle/P_H$ is isomorphic with $N/P_{H\cap N}$. If N is also reflexive in S, then N/P_H is a normal subgroup of S/P_H and $(S/P_H)/(N/P_H)$ is isomorphic with S/P_N (Theorem 1.5 of [2]).

The mentioned results suggest that the simple reflexive unitary subsemigroups of semigroups can play a similar role to the normal subgroups of groups. But, as the following example shows, it is necessary to make the conditions stronger. Let $S_1(\ominus)$ and $S_2(+)$ be (completely) simple semigroups with $S_1\cap S_2=\phi$. Let 0 denote a symbol, $0\notin S_1$ and $0\notin S_2$. On the set $S=S_1\cup S_2\cup\{0\}$, we define an operation. For every $t, s\in S$, let

$$ts=\begin{cases} t\ominus s & \text{if } t, s\in S_1, \\ t+s & \text{if } t, s\in S_2, \\ 0 & \text{in other cases.}\end{cases}$$

It can be easily verified that S_1 and S_2 are (completely) simple unitary subsemigroups of S and $S_1S_2=S_2S_1=0$ is not unitary in S. We note that $\langle S_1, S_2\rangle=S\neq S_1S_2$.

Denote $U(S)$ the set of those unitary subsemigroups of the semigroup S all of whose unitary subsemigroups are simple. As in [2], we say that a semigroup H of S is an *n-unitary subsemigroup* of S if

(a) $H\in U(S)$ and H is reflexive in S,

(b) $V\in U(S)$ implies $\langle H, V\rangle=HV\in U(S)$.

In [2] it is shown that

Lemma 1. *If $H\subseteq N$ are n-unitary subsemigroups of a semigroup,*