4. An Application of the Perturbation Theorem for m-Accretive Operators. II

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(Communicated by Kôsaku YOSIDA, M. J. A., Jan. 12, 1984)

1. Introduction and statement of the result. This note is concerned with the homogeneous Dirichlet problem for a nonlinear elliptic equation

$$(1)\qquad -\sum_{j=1}^{n}\frac{\partial}{\partial x_j}\left(\left|\frac{\partial u}{\partial x_j}\right|^{p-2}\frac{\partial u}{\partial x_j}\right)+\beta(x,u)=f \qquad \text{on } \Omega,$$

where Ω is a bounded domain in $\boldsymbol{R}^n$ with smooth boundary.

Let $W_0^{1,p}(\Omega)$ be the usual Sobolev space. We consider only real-valued functions in the case of $p\geqq 2$. Then it follows from the Poincaré inequality that $W_0^{1,p}(\Omega)\subset L^2(\Omega)$. Setting

$$\phi(u)=\frac{1}{p}\sum_{j=1}^{n}\int_{\Omega}\left|\frac{\partial u}{\partial x_j}\right|^{p}dx \qquad \text{for } u\in W_0^{1,p}(\Omega)$$

and $\phi(u)=+\infty$ otherwise, ϕ is a proper lower semicontinuous convex function on $L^2(\Omega)$. The subdifferential $\partial\phi$ of ϕ is given by

$$\partial\phi(u)=-\sum_{j=1}^{n}\frac{\partial}{\partial x_j}\left(\left|\frac{\partial u}{\partial x_j}\right|^{p-2}\frac{\partial u}{\partial x_j}\right) \qquad \text{for } u\in D(\partial\phi)\subset W_0^{1,p}(\Omega)$$

and is m-accretive in $L^2(\Omega)$ (see e.g. [1] or [2]).

Let $\beta\in C^1(\Omega\times J)$, where J is an open interval on $\boldsymbol{R}$ containing the origin. We assume that

(i) $\beta(x,0)=0$ for every $x\in\Omega$, and $\partial\beta/\partial s\geqq 0$ on $\Omega\times J$.

(ii) for every $x\in\Omega$, $\beta(x,\cdot):J\to\boldsymbol{R}$ is onto.

Then we can introduce the m-accretive operator $\tilde\beta$ in $L^2(\Omega)$:

$$D(\tilde\beta)=\{u\in L^2(\Omega)\,;\,u(x)\in J \text{ (a.e. on } \Omega),\ \beta(x,u(x))\in L^2(\Omega)\},$$
$$\tilde\beta u(x)=\beta(x,u(x)) \qquad \text{for } u\in D(\tilde\beta).$$

The purpose of this note is to prove the following

Theorem 1. *Let $A=\partial\phi$ and $B=\tilde\beta$ be m-accretive operators as above. Assume that there are nonnegative constants c, a and b $[b<p^p(p-1)^{-(p-1)}]$ such that on $\Omega\times J$*

$$(2)\qquad \sum_{j=1}^{n}\left|\frac{\partial\beta}{\partial x_j}(x,s)\right|^{p}\leqq\{c+as^2+b[\beta(x,s)]^2\}\left[\frac{\partial\beta}{\partial s}(x,s)\right]^{p-1}.$$

Then $A+B=\partial\phi+\tilde\beta$ with domain $D(A)\cap D(B)$ is m-accretive in $L^2(\Omega)$.

Noting that $A=\partial\phi$ is strictly accretive (for a precise estimate see Simon [7]), we obtain

Corollary 2. *For every $f\in L^2(\Omega)$ there exists a unique solution $u\in D(\partial\phi)\cap D(\tilde\beta)$ of the equation* (1).