1. A Short Proof of a Theorem Concerning Homeomorphisms of the Unit Circle*)

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1. In [4], Rieffel classified the C^*-algebras associated with irrational rotations on the unit circle S^1 in the complex plane. Recently these C^*-algebras have played an important rôle in the theory of operator algebras.

The author and Takemoto [1] extended the Rieffel's result to the case of C^*-algebras associated with monothetic compact abelian groups. A compact abelian group G is said to be monothetic if there exists a homomorphism from the group Z of all integers to a dense subgroup of G (cf. [5, 2.3]). In [1] and [2], we considered more general cases. Namely, we studied the C^*-algebras associated with topologically transitive compact dynamical systems. A dynamical system (Ω, σ) is said to be topologically transitive if the homeomorphism σ admits a point ω in the compact space Ω such that the orbit $O(\omega)$ $(=\{\sigma^n(\omega): n \in Z\})$ is dense in Ω (cf. [6, 5.4]). So we are interested in the existence and the classification of such dynamical systems. In case $\Omega = S^1$, every topologically transitive homeomorphism σ is conjugate to an irrational rotation. It is well-known that this theorem was established by Poincaré [2]. Nowadays we can see several kinds of proofs in many books, in which the rotation number of σ plays an important rôle. In this note, we give a short and elementary proof without rotation numbers.

2. Two homeomorphisms σ_1 and σ_2 of S^1 are said to be conjugate if there exists a homeomorphisms h on S^1 such that $\sigma_1 = h\sigma_2 h^{-1}$. For a real number θ, we denote by R_θ the rotation: $R_\theta(e^{2\pi i x}) = e^{2\pi i (x+\theta)}$ on S^1. We shall prove the following equivalences.

Theorem. *Let σ be a homeomorphism of S^1. Then the following statements are equivalent;*

(1) *$O(z)$ is dense in S^1 for some z in S^1,*

(2) *$O(z)$ is dense in S^1 for every z in S^1,*

(3) *σ is conjugate to R_θ for some irrational number θ* $(0<\theta<1/2)$.

When σ satisfies the condition (1) *or* (2), *the rotation R_θ in* (3) *is uniquely determined.*

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