29. *A Note on the Fundamental Group of a Unirational Variety*

By Noriyuki SUWA
Department of Mathematics, University of Tokyo

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1. Introduction. Let k be an algebraically closed field and let X be a smooth projective variety over k. X is unirational (or separably unirational) if there is a dominant rational map $\boldsymbol{P} \dashrightarrow X$ where $\boldsymbol{P}$ is a projective space such that the extension of fields $k(\boldsymbol{P})/k(X)$ is finite (or finite separable). Serre showed in [5] the following results.

(1) An étale covering of a unirational (or separably unirational) variety is also unirational (or separably unirational).

(2) The fundamental group of a unirational variety is finite.

(3) If k is of characteristic 0, every unirational variety is simply connected.

Further the following facts are known about the fundamental variety in the case of characteristic $p>0$.

(4) If X is separably unirational and of dimension 3, then X is simply connected (Nygaard [4]).

(5) If X is unirational and of dimension ≤ 3, $\pi_1(X)$ is p-torsion-free (Katsura [3]*), Crew [1]).

(6) The order of the fundamental groups of unirational surfaces are not bounded (Shioda [6], remark 7).

In this note we will show the following:

Theorem. *Let k be an algebraically closed field of characteristic $p>0$ and X a separably unirational variety over k. Then the fundamental group $\pi_1(X)$ of X is p-torsion-free.*

2. Proof of the theorem. The proof is based on the theory of de Rham-Witt complex of Deligne-Illusie [2] and a recent result of Crew [1]. We follow the notation of [2].

Proof. Since X is separably unirational, we have

$$H^0(X, \Omega_X^i)=0 \qquad \text{for } i>0.$$

Now the isomorphism

$$W.\Omega_X^i/VW.\Omega_X^i \xrightarrow{\sim} Z.\Omega_X^i$$

induces the isomorphism

$$H^0(X, W\Omega_X^i/VW\Omega_X^i) \xrightarrow{\sim} \varprojlim_{C} H^0(X, Z_n\Omega_X^i)$$

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*) Katsura has communicated to me orally that the method in [3] is also valid for unirational three-folds.