# 28. An Algebraic Computation of the Alexander Polynomial of a Plane Algebraic Curve

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**1. Introduction and the statement of the result.** Let $C$ be the algebraic curve in $\boldsymbol{C}^2$ defined by a reduced polynomial $f$. We denote by $\Omega^{\cdot}_{C^2}(*C)$ the algebra of the rational differential forms on $\boldsymbol{C}^2$ which are holomorphic on the complement $X=\boldsymbol{C}^2-C$. Let $\nabla_\alpha:\Omega^j_{C^2}(*C)\to\Omega^{j+1}_{C^2}(*C)$ be the regular connection in the sense of Deligne ([1]) defined by $\nabla_\alpha\varphi=d\varphi+\alpha d\log f\wedge\varphi$ with a real number $\alpha$. We denote by $H^j(\Omega^{\cdot}_{C^2}(*C),\nabla_\alpha)$ the $j$-th cohomology group of the de Rham complex

$$\cdots\longrightarrow\Omega^j_{C^2}(*C)\xrightarrow{\nabla_\alpha}\Omega^{j+1}_{C^2}(*C)\longrightarrow\cdots.$$

In [5] A. Libgober defined the Alexander polynomial of a plane algebraic curve. Let us review the definition.

**Definition 1.1.** Let $\bar{C}$ be an irreducible algebraic curve in $\boldsymbol{P}^2$. We take a complex line $H_\infty$ such that $H_\infty$ intersects $\bar{C}$ transversally. Let $C$ denote $\bar{C}\cap(\boldsymbol{P}^2-H_\infty)$ and let $X$ be the complement of $C$ in $\boldsymbol{P}^2-H_\infty$.

Let $p:X^{ab}\to X$ be an infinite cyclic covering of $X$. Then the ring of the Laurent polynomials $\boldsymbol{C}[t^{-1},t]=\Lambda$ operates on $H^1(X^{ab};\boldsymbol{C})$ by means of the deck transformations. The $\Lambda$-module $H^1(X^{ab};\boldsymbol{C})$ has a presentation of the form

$$\Lambda/_{(f_1(t))}\oplus\cdots\oplus\Lambda/_{(f_k(t))}$$

with some polynomials $f_1(t),\cdots,f_k(t)$. We call the product $\prod_{j=1}^k f_j(t)$ the *Alexander polynomial* of $\bar{C}$ (or $C$).

**Remarks 1.2.** i) In the proof of Theorem (1.3), we show that $\dim_C H^1(X^{ab};\boldsymbol{C})$ is finite.

ii) The Alexander polynomial of the curve $C$ is determined up to unit and does not depend on the choice of a line $H_\infty$.

We have the following

**Theorem 1.3.** *Let $C\cap\boldsymbol{C}^2$ be an irreducible algebraic curve which intersects transversally with the line at infinity. Let $h_\alpha$ denote $\dim_C H^1(\Omega^{\cdot}_{C^2}(*C,\nabla_\alpha)$ for a real number $\alpha$. Let $\Delta_C(t)$ be the Alexander polynomial of $C$. Then we have*

$$\Delta_C(t)=\prod_{0<\alpha<1}(t-\exp 2\pi\sqrt{-1}\,\alpha)^{h_\alpha}.$$

*Moreover the numbers $\alpha$ with $h_\alpha\neq 0$ are rational numbers with $n\alpha\in\boldsymbol{Z}$, where we denote by $n$ the degree of our curve $C$.*

**2. Proof of the theorem.** Let $\bar{C}$ be the algebraic closure of $C$