25. *Iterated Log Type Strong Limit Theorems for Self-Similar Processes*

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1. Let $\{X(t, \omega); 0 \leq t < +\infty\}$ be a real valued separable, measurable, stochastically continuous self-similar process of order $H>0$, where "self-similar process" means that for any $a>0$, $\{X(t)\}$ and $\{a^{-H}X(at)\}$ have the same finite dimensional distribution. We will denote it by $X(t) \stackrel{d}{=} a^{-H}X(at)$. Set

$$Y(\omega) = \sup_{0 \leq t \leq 1} |X(t, \omega)|.$$

Theorem 1. *Let $f(x)$ be a positive, continuous, non-decreasing function defined on the positive half line. Assume that $E[f(Y)]$ is finite. Let $\phi(x)$ be a positive, continuous function defined on the positive half line which satisfies the following conditions;*

(i) *$\phi(x)$ is non-decreasing,*

(ii) $\lim_{x \downarrow 1} \sup_{n=1,2,\cdots} \phi(x^n)/\phi(x^{n-1}) = c < +\infty,$

and

(iii) $\int^{+\infty} (xf(\phi(x)))^{-1} dx < +\infty.$

Then, we have

$$\overline{\lim_{s \to +\infty}} \frac{|X(s, \omega)|}{s^H \phi(s)} \leq c \qquad \text{a.s.}$$

Theorem 2. *Let $g(x)$ be a positive, continuous, non-increasing function defined on the positive half line. Assume that $E[g(Y)]$ is finite. Let $\psi(x)$ be a positive, continuous function defined on the positive half line which satisfies the following conditions;*

(i) *$\psi(x)$ is non-increasing,*

and

(ii) $\int^{+\infty} (xg(\psi(x)))^{-1} dx < +\infty.$

Then, we have

$$\underline{\lim}_{s \to +\infty} \frac{\sup_{0 \leq t \leq s} |X(t, \omega)|}{s^H \psi(s)} \geq 1 \qquad \text{a.s.}$$

2. First, we prove the following

Lemma 1. *If $E[f(Y)] = K < +\infty$, then for $x>0$, we have*

$$P\Big(\sup_{0 \leq t \leq \lambda} |X(t, \omega)| \geq x\Big) \leq K/f(\lambda^{-H}x).$$