23. The Exponential Calculus of Microdifferential Operators of Infinite Order. III

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1. **Introduction.** In this note we calculate r and c satisfying

$$:ae^p: :be^q: = :ce^r:. \tag{1.1}$$

Here a, b, p, q are given formal symbols (see [1]–[4] for the notation). When $a=b=1$, p and q are symbols, such r and c are computed in [2] (cf. [3], [4]). In our present formula, we can take a, b, p, and q to be formal symbols, that is, infinite sums of symbols which satisfy some conditions.

2. **Double formal symbols.** Let X be an open set in $C^n = \{x=(x_1, \cdots, x_n); x_j \in C, 1 \leq j \leq n\}$, $\mathring{x}^*$ a point in the cotangent bundle $T^*X \simeq X \times C^n = \{(x, \xi) \in X \times C^n\}$ of X.

Definition 1. Let Ω be a conic neighborhood of $\mathring{x}^*$ in T^*X. Let

$$P(t_1, t_2; x, \xi) = \sum_{j,k=0}^{\infty} t_1^j t_2^k P_{jk}(x, \xi) \tag{2.1}$$

be a formal power series in (t_1, t_2) with coefficients $P_{jk}(x, \xi)$ ($j, k=0, 1, 2, \cdots$) holomorphic in Ω. The formal series $P(t_1, t_2; x, \xi)$ is said to be a double formal symbol defined in Ω if for any $\Omega' \subset\subset \Omega$ there exist constants d, A which satisfy the following conditions:

(a) $0<d$, $0<A<1$.

(b) For each $h>0$ there is a constant $C>0$ such that

$$|P_{jk}(x, \xi)| \leq CA^{j+k} \exp(h|\xi|) \tag{2.2}$$

for all $j, k=0, 1, 2, \cdots$; $(x, \xi) \in \Omega'$ satisfying $|\xi| \geq (j+k+1)d$.

The space of all double formal symbols defined in Ω is denoted by $\hat{S}_2(\Omega)$, which is a commutative ring under the addition and the product to be those of formal power series. Set $\hat{S}(\Omega) = \hat{S}_1(\Omega) = \hat{S}_2(\Omega)|_{t_2=0}$, then $\hat{S}(\Omega)$ is the ring of all formal symbols defined in Ω ([2], Def. 1; here we consider $t=t_1$).

Definition 2. A double formal symbol

$$P(t_1, t_2; x, \xi) = \textstyle\sum_{j,k=0}^{\infty} t_1^j t_2^k P_{jk}(x, \xi)$$

defined in Ω is said to be equivalent to zero and is written $P(t_1, t_2; x, \xi) \sim 0$ if for any $\Omega' \subset\subset \Omega$ there exist constants d, A which satisfy the following conditions:

(a) $0<d$, $0<A<1$.

(b) For each $h>0$ there is a constant $C>0$ such that