21. The Nonrelativistic Limit of Modified Wave Operators for Dirac Operators

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We shall consider the Dirac operator

$$L(c)=c\sum_{j=1}^{3}\alpha_j D_j+c^2\beta+V(x)\qquad\left(x\in \boldsymbol{R}^3,\ D_j=\frac{1}{i}\frac{\partial}{\partial x_j}\right),$$

where $c>0$ is the velocity of light and α_j, β are 4×4 matrices given by

$$\alpha_1=\begin{bmatrix} & & & 1\\ & & 1 & \\ & 1 & & \\ 1 & & & \end{bmatrix},\quad \alpha_2=\begin{bmatrix} & & & -i\\ & & i & \\ & -i & & \\ i & & & \end{bmatrix},\quad \alpha_3=\begin{bmatrix} & & 1 & \\ & & & -1\\ 1 & & & \\ & -1 & & \end{bmatrix},$$

$$\beta=\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1\end{bmatrix},$$

which satisfy the anti-commutation relation $\alpha_j\alpha_k+\alpha_k\alpha_j=2\delta_{jk}I$ $(j, k=1, 2, 3, 4)$ with $\alpha_4=\beta$ (I is the 4×4 unit matrix). The scalar potential $V(x)$ is assumed to satisfy the following *condition* (A); *there exist positive constants* $0<\delta$ (≤ 1), $e>0$ *and a positive integer* $m\geq 3$ *such that*

(A-1) $\quad m=3$ *if* $\delta>\dfrac{1}{2}$ *and* $m=\left[\dfrac{1}{\delta}\right]+3$ *if* $0<\delta\leq\dfrac{1}{2}$,

and $V(x)$ *is a real-valued* C^m*-function in* $\boldsymbol{R}^3\backslash 0$ *satisfying*

(A-2) $\quad D^\alpha V(x)=O(|x|^{-|\alpha|-\delta})$ *as* $|x|\to\infty$ $\quad(|\alpha|\leq m)$,

(A-3) $\quad |V(x)|\leq\dfrac{e}{r}$ $\quad(0<r\leq 1)$,

where $D^\alpha=D_1^{\alpha_1}D_2^{\alpha_2}D_3^{\alpha_3}$, $|\alpha|=\alpha_1+\alpha_2+\alpha_3$ *for a multi-index* $\alpha=(\alpha_1, \alpha_2, \alpha_3)\in \boldsymbol{Z}^3$ $(\alpha_j\geq 0)$.

It is evident that $L(c)$ is formally selfadjoint in the Hilbert space $\mathcal{L}^2=[L^2(\boldsymbol{R}^3)]^4$. A symmetric operator $L(c)$ defined on $[C_0^\infty(\boldsymbol{R}^3)]^4$ has the (not necessarily unique) selfadjoint extension[1], and is essentially selfadjoint if $c>2e$ (see Kato [7, Chapter V, § 4] and note that Arai [3] proposes a refined result). We denote by $H_0(c)$ the unperturbed selfadjoint operator with $V(x)\equiv 0$.

Let $H_0(c)=\int_{-\infty}^{\infty}\lambda dE^{(c)}(\lambda)$ be the spectral representation of $H_0(c)$. It

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[1] This fact will be also proved elsewhere.