2. On an Elaboration of M. Kac's Theorem Concerning Eigenvalues of $-\Delta$ in a Region with Randomly Distributed Small Obstacles

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Let Ω be a bounded domain in $\boldsymbol{R}^3$ with smooth boundary γ. Let $0\leq\mu_1(\varepsilon;w(m))\leq\mu_2(\varepsilon;w(m))\leq\cdots$ be the eigenvalues of $-\Delta(=-\mathrm{div}\ \mathrm{grad})$ in $\Omega_{\varepsilon,w(m)}=\Omega\setminus\overline{\bigcup_{i=1}^m B(\varepsilon;w_i^{(m)})}$ under the Dirichlet condition on its boundary. We arrange them repeatedly according to their multiplicities. Here $B(\varepsilon;w)=\{x\in\boldsymbol{R}^3;|x-w|<\varepsilon\}$ and $w(m)$ denotes the set of m-points $\bigcup_{i=1}^m\{w_i^{(m)}\}$. Let $V(x)$ be C^1 function on $\bar{\Omega}$ satisfying $V(x)\geq 0$ and

$$\int_\Omega V(x)dx=1.$$

We consider Ω as the probability space with probability density $V(x)dx$.

Kac's theorem is the following

Theorem (Kac [1], Rauch-Taylor [5]). *Fix k and $\alpha>0$. Then*

$$\lim_{m\to\infty}\boldsymbol{P}(w(m)\in\Omega^m;|\mu_k(\alpha/m;w(m))-\mu_k^V|<\varepsilon)=1$$

for any $\varepsilon>0$. That is, $\mu_k(\alpha/m;w(m))$ tends to μ_k^V in probability. Here μ_k^V is the k-th eigenvalue of $-\Delta+4\pi\alpha V(x)$ in Ω under the Dirichlet condition on γ.

In this note we give an elaboration of Kac's theorem. We have the following

Theorem 1. *Fix k and $\alpha>0$. Then*

$$\lim_{m\to\infty}\boldsymbol{P}(w(m)\in\Omega^m;|\mu_k(\alpha/m;w(m))-\mu_k^V|<\varepsilon m^{-\beta})=1$$

for any $\varepsilon>0$ and any fixed $\beta\in[0,1/4)$.

Remark. Kac [1] proved his result by using the theory of Wiener sausage in case $V(x)=(\text{volume of }\Omega)^{-1}$. After Kac [1], Rauch-Taylor [5] gave the result for general $V(x)$ by combining functional analysis of operators and the Feynmann-Kac formula. See also Simon [6], Papanicolaou-Varadhan [4].

Our proof of Theorem 1 is quite different from [1], [5]. The main idea is to use perturbational calculus using Green's function of $-\Delta$. A direct construction of an approximate Green's function of $-\Delta$ in $\Omega_{\alpha/m,w(m)}$ under the Dirichlet condition on $\partial\Omega_{\alpha/m,w(m)}$ in terms of Green's function of $-\Delta$ in Ω under the Dirichlet condition on γ enables us to give a remainder estimate in Theorem 1. For the method using