139. On the Cone of Curves of Algebraic Varieties

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In this paper we announce a structure theorem on the cone of curves of algebraic varieties defined over a field of characteristic zero. Details will appear elsewhere. This theorem should be one of the key steps toward the theory of minimal models of algebraic varieties. We already have the so-called contraction theorem (Theorem 4), which is a generalization of Castelnuovo's criterion of exceptional curves of the first kind. Our weak cone theorem guarantees the existence of a good extremal ray to be contracted if the model is not minimal. The remaining thing to be proved would be the theorem on elementary transformations (see Reid [5], [6], Kawamata [2]).

1. We fix our notation. Let X be a normal projective variety. We define: $N_1(X)=\{\text{1-cycles on } X\}/\approx\otimes\boldsymbol{R}$, $N^1_{\boldsymbol{Q}}(X)=\{\text{line bundles on } X\}/\approx\otimes\boldsymbol{Q}$, $N^1(X)=N^1_{\boldsymbol{Q}}(X)\otimes\boldsymbol{R}$, and $\overline{NE}(X)=$ the closed convex cone in $N_1(X)$ generated by effective 1-cycles, where $\approx$ denotes numerical equivalence. $N_1(X)$ and $N^1(X)$ are dual to each other by intersection pairing. An element $D\in N^1(X)$ is called "nef" (numerically effective or numerically semipositive) if $D\geqq 0$ on $\overline{NE}(X)$. Moreover, if $(D^n)>0$ with $n=\dim X$, then D is called "big". We denote by $\mathrm{Div}\,(X)$ the group of Weil divisors on X and $K_X\in\mathrm{Div}\,(X)$ the canonical divisor on X. A "$\boldsymbol{Q}$-divisor" is an element $D\in\mathrm{Div}\,(X)\otimes\boldsymbol{Q}$. D is called "$\boldsymbol{Q}$-Cartier" if there is a positive integer a such that aD is a Cartier divisor. If K_X is $\boldsymbol{Q}$-Cartier, X is called "$\boldsymbol{Q}$-Gorenstein". For $\boldsymbol{Q}$-Cartier divisors we can define intersection numbers with 1-cycles by linearity and thus their numerical classes in $N^1(X)$. For $r\in\boldsymbol{R}$ we define $\ulcorner r\urcorner=\min\{t\in\boldsymbol{Z};\ t\geqq r\}$ and $\{r\}=r+\ulcorner -r\urcorner$. Let $D=\sum a_iD_i$ be a $\boldsymbol{Q}$-divisor, where $a_i\in\boldsymbol{Q}$ and the D_i are mutually distinct prime divisors. We define $\ulcorner D\urcorner=\sum\ulcorner a_i\urcorner D_i$ and $\{D\}=\sum\{a_i\}D_i$. X is said to have only "canonical singularities" if X is $\boldsymbol{Q}$-Gorenstein and if the following condition is satisfied: there is a resolution of singularities $f: Y\to X$ such that $aK_Y=f^*(aK_X)+a\sum a_iE_i$ with $a_i\geqq 0$, where a is a positive integer such that aK_X is a Cartier divisor and the E_i are exceptional divisors.

Theorem 1 (Kawamata [1] or Viehweg [8]). *Let X be a non-singular projective variety and $D\in\mathrm{Div}\,(X)\otimes\boldsymbol{Q}$. Assume the following conditions:*

(i) *D is nef and big and*