138. Deformations of Complements of Lines in P^2

By Hironobu MAEDA
Department of Mathematics, University of Tokyo

(Communicated by Kunihiko KODAIRA, M. J. A., Dec. 12, 1983)

§1. **Introduction.** In this paper we shall study deformations of complements of lines in P^2, based on the theory of logarithmic deformation introduced by Kawamata ([2]). The result is that the standard completion (see below) of complements of lines in P^2 has smooth versal family of logarithmic deformations. This provides examples of surfaces of logarithmic general type with unobstructed deformations even though $H^2(X, \Theta(\log D)) \neq 0$.

Let $\Delta_1, \cdots, \Delta_n$ be projective lines on a complex projective plane P^2, where $\Delta_i \neq \Delta_j$ for $i \neq j$, and let $\Delta = \bigcup_i \Delta_i$. We call $P \in \Delta$ a higher multiple point of Δ, if the multiplicity of Δ at P is greater than two. Let $P_1, \cdots, P_s$ be all the higher multiple points of Δ with respective multiplicities $\nu_1, \cdots, \nu_s$. Let $\mu_1, \cdots, \mu_n$ be the numbers of higher multiple points lying over $\Delta_1, \cdots, \Delta_n$, respectively. Blowing up P^2 with center at $C = P_1 + \cdots + P_s$, we obtain a complete non-singular surface X and a birational morphism $\mu: X \to P^2$. Let $E_j = \mu^{-1}(P_j)$, Δ^* the proper transform of Δ and D the set-theoretical inverse image of Δ, i.e. $D = \mu^{-1}(\Delta) = \Delta^* + \Sigma_j E_j$. Then D is a divisor on X with normal crossings. The non-singular triple $(X \setminus D, X, D)$ is called the standard completion of $P^2 \setminus \Delta$ (cf. [1, p. 4]) and can be used as a substitute for the complement of lines Δ in P^2.

For the definition of the family of logarithmic deformations of non-singular triple, we refer to [2].

Then we have the following

Theorem. (1) *For any choice of Δ, the non-singular triple $\xi = (X \setminus D, X, D)$ has no obstruction to logarithmic deformations.*

(2) *The numbers $h^i = \dim H^i(X, \Theta(\log D))$ are computed and classified according to the type* (cf. [1, Table]) *of Δ as following* Table I.

(3) *If Δ corresponds to the configurations of Pappus or Desargues* (Fig. 1), *then we get examples with $H^2(X, \Theta(\log D)) \neq 0$.*

(4) *There exists an infinite series of Δ's of type* III *with* $H^1(X, \Theta(\log D)) = 0$.

In this paper we outline a proof of (1). For the details we refer to [3].

§2. **Unobstructedness of $(X \setminus D, X, D)$.** Let $(\hat{\mathcal{X}} \setminus \hat{\mathcal{D}}, \hat{\mathcal{X}}, \hat{\mathcal{D}}, \hat{\pi}, \hat{B})$ be the versal family of logarithmic deformations of $(X \setminus D, X, D)$ con-