15. Branching of Singularities for Degenerate Hyperbolic Operator and Stokes Phenomena. IV

By Kazuo AMANO*) and Gen NAKAMURA**)

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1. This note is a continuation of our previous notes [2]–[4]. The aims of this note are to generalize the results of Alinhac [1], Hanges [6] and Taniguchi-Tozaki [8] concerning the sufficient condition of branching of singularities for more general second order differential equations with higher order degeneracy and to show the computability of our condition. The details and further discussions will appear in [5].

2. **Review of [4] and results.** Let $t\in[-T,T]$, $x=(x_1,\cdots,x_n)\in R^n$, $D_t=\partial/(\sqrt{-1}\,\partial x_j)$ $(1\leq j\leq n)$ and $P=P(t,X,D_t,D_x)$ be a second order linear partial differential operator of the form:

$$P=\sum_{j=0}^{2}\sum_{i=0}^{2-j}P_{i,j}(t,X,D_x)D_t^{2-j-i}$$

where each $P_{i,j}(t,x,\xi)$ is a homogeneous polynomial of degree i with respect to $\xi=(\xi_1,\cdots,\xi_n)\in R^n$. For simplicity, we assume all the coefficients have bounded derivatives of any order on $[-T,T]\times R^n_x$.

We assume the following conditions (A.1)–(A.3) for P which are invariant under a change of x variable.

(A.1) $P_2(t,x,\tau,\xi)$ is smoothly factorizable as follows:

$$P_2(t,x,\tau,\xi)=\prod_{j=0}^{2}(\tau-t^l\lambda_j(t,x,\xi))$$

where $l\in N$ and $\lambda_j(t,x,\xi)\in C^\infty([-T,T]\times R^n\times(R^n_\xi-\{0\}))$ $(j=1,2)$ are real valued.

(A.2) There exists a constant $c>0$ such that

$$|\lambda_1(t,x,\xi)-\lambda_2(t,x,\xi)|\geq c|\xi| \qquad \text{for any } (t,x,\xi).$$

(A.3) Each $P_{i,j}(t,x,\xi)$ $(il\geq j,\ 2-j-i\geq 0)$ has the property:

$$P_{i,j}(t,x,\xi)=t^{il-j}\tilde{P}_{i,j}(t,x,\xi)$$

where $P_{i,j}(t,x,\xi)$ is a homogeneous polynomial of degree i in ξ and its coefficients have bounded derivatives of any order on $[-T,T]\times R^n_x$.

Now, set $L_0=D_t^2+t^lP_{1,0}(0,x,\xi)D_t+t^{l-1}P_{1,1}(0,x,\xi)+t^{2l}P_{2,0}(0,x,\xi)$ and denote the central connection coefficients associated to L_0 by $T_\pm^{(i,j)}(x,\xi)$ $(i,j=1,2)$ (see [4] for the definition). Also, denote the (i,j)-cofactor of the matrix $\left(T_\pm^{(i,j)}(x,\xi)\,;\begin{smallmatrix}i\downarrow 1,2\\ j\to 1,2\end{smallmatrix}\right)$ by $T_\pm^{(i,j)}(x,\xi)$. Moreover, we denote

*) Department of Mathematics, Josai University.

**) Department of Mathematics, M.I.T. and Department of Mathematics, Josai University.