133. *Semivariation and Operator Semivariation of Hilbert Space Valued Measures*

By Yûichirô KAKIHARA
Department of Mathematical Science, College of Science and Engineering, Tokyo Denki University

(Communicated by Kôsaku YOSIDA, M. J. A., Dec. 12, 1983)

§1. **Introduction.** Let $\mathfrak{H}$ and $\mathfrak{K}$ be a pair of Hilbert spaces. $B(\mathfrak{H})$ denotes the Banach space of all bounded linear operators on $\mathfrak{H}$ with the identity 1 and the uniform norm $\|\cdot\|$, and $T(\mathfrak{H})$ denotes the set of all trace class operators on $\mathfrak{H}$ with the trace $\mathrm{Tr}\,(\cdot)$ and the trace norm $\|\cdot\|_\tau$. Let $X=S(\mathfrak{H},\mathfrak{K})$ be the set of all Hilbert-Schmidt class operators from $\mathfrak{H}$ into $\mathfrak{K}$. For $x, y\in X$ define $[x,y]=x^*y\in T(\mathfrak{H})$, $\langle x,y\rangle_X=\mathrm{Tr}\,[x,y]$ and $\|x\|_X=\langle x,x\rangle_X^{1/2}$. Then X becomes a Hilbert space with the inner product $\langle\cdot,\cdot\rangle_X$.

Let $(\Omega,\mathfrak{F})$ be a measurable space. We consider X-valued measures defined on $\mathfrak{F}$. Denote by $ca(\Omega\,;X)$ the set of all X-valued bounded and countably additive, in the norm $\|\cdot\|_X$, measures on $\mathfrak{F}$. The *operator semivariation* of $\xi\in ca(\Omega\,;X)$ is the function $\|\xi\|_o(\cdot)$ whose value on a set $A\in\mathfrak{F}$ is given by

$$\|\xi\|_o(A)=\sup\left\|\sum_{k=1}^n \xi(A_k)a_k\right\|_X \tag{1.1}$$

where the supremum is taken for all finite measurable partitions $\{A_1,\cdots,A_n\}$ of A and for all finite subsets $\{a_1,\cdots,a_n\}\subset B(\mathfrak{H})$ with $\|a_k\|\leqslant 1$, $1\leqslant k\leqslant n$ (cf. Kakihara [2, Definition 3.1 and §5]). The *semivariation* of ξ is the function $\|\xi\|(\cdot)$ whose value on a set $A\in\mathfrak{F}$ is given in (1.1) by replacing $a_k\in B(\mathfrak{H})$ with $\lambda_k\in C$ (the complex number field) such that $|\lambda_k|\leqslant 1$, $1\leqslant k\leqslant n$ (cf. Diestel and Uhl [1, pp. 2–4]). Then we have $\|\xi(A)\|_X\leqslant\|\xi\|(A)\leqslant\|\xi\|_o(A)$, $A\in\mathfrak{F}$. In §2, we shall obtain the characterization of those measures $\xi\in ca(\Omega\,;X)$ for which the following condition is satisfied:

$$\|\xi\|_o(A)=\|\xi(A)\|_X,\qquad A\in\mathfrak{F}. \tag{1.2}$$

In §3, we consider the set $ca(\Omega\,;\mathfrak{H})$ of all $\mathfrak{H}$-valued bounded and countably additive measures on $\mathfrak{F}$. The *operator semivariation* of $\xi\in ca(\Omega\,;\mathfrak{H})$ is the function $\|\xi\|_o(\cdot)$ whose value on a set $A\in\mathfrak{F}$ is given by

$$\|\xi\|_o(A)=\sup\left\|\sum_{k=1}^n a_k\xi(A_k)\right\| \tag{1.3}$$

where the supremum is taken as in the case of (1.1) and $\|\cdot\|$ is the norm of $\mathfrak{H}$. If we identify $\mathfrak{H}$ with $S(\mathfrak{H},C)$, we see that the operator semi-