130. *Fundamental Theorems in Global Knot Theory.* I

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1. Introduction. A *knot* K in an m-dimensional smooth manifold M^m is a submanifold of M^m diffeomorphic to the $(m-2)$-sphere S^{m-2}. The knots in manifolds are the object of global knot theory which is a generalization of the knot theory in spheres. Each manifold has its own knot theory (see [6, Theorem 1]).

A knot K in M^m is said to be *unknotted* if K bounds an $(m-1)$-disk smoothly imbedded in M^m. A knot K in M^m is said to be *local* if there exists an m-disk D^m smoothly imbedded in M^m such that $D^m \supset K$.

Criterion theorems of unknottedness and localness for knots in manifolds are fundamental in global knot theory as the unknotting theorem of Papakyriakopoulos and that of Levine are fundamental in the classical knot theory and the higher dimensional knot theory ([3], [2]).

In this note we announce the unknotting and localness theorems for knots in highly connected smooth manifolds by means of knot modules (Theorems 3 and 4), generalizing the unknotting theorem by Levine. As an application the localness and the unknottedness of genus 1 knots in $S^n \times S^{n+1}$ will be determined by computing their knot modules ([6, Theorem 1]).

2. Seifert surfaces. Let K be a knot in a connected m-dimensional smooth manifold M^m $(m \geqq 3)$. A compact connected $(m-1)$-dimensional submanifold V of M^m such that $\partial V = K$ is called a *Seifert surface* for K if V is transversally orientable.

Proposition 1. *Let M^m be a connected closed m-dimensional smooth manifold such that $m \geqq 4$ and $H^2(M^m; \boldsymbol{Z}) = 0$, and let K be a knot in M^m. Then there exists a Seifert surface for K.*

A knot K in M^m is said to be *r-simple* if the homotopy groups $\pi_i(X)$ of the complement $X = M^m - K$ are as follows:

$$\pi_1(X) \cong \boldsymbol{Z}, \quad \pi_i(X) = 0 \qquad \text{for } 2 \leqq i \leqq r.$$

Proposition 2. *Let M^m be a q-connected closed m-dimensional smooth manifold and let K be a knot in M^m, where $m \geqq 6$, $2 \leqq q \leqq [m/2] - 1$.*

(a) *If K is k-simple for* $1 \leqq k \leqq q$ (resp. $1 \leqq k \leqq q-1$) *in case m is odd* (resp. m is even), *then there exists a Seifert surface for K which is k-connected.*